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Robust and flexible planning of power system generation capacity

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Robust and flexible planning of power system generation capacity

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ABSTRACT

The successful evolution of a power system is achieved when its future growth path is visualized. Visualizing and interpreting the future are crucial to understand the risks to which the power system is exposed. These are mostly caused by the interdependencies between the power system and other systems (e.g., transportation sector, fuels sector, industry, etc.); and the resulting uncertain environment where these systems perform. Then, the objectives of planning are to reduce the risks of uncertainties and to gain some control over the future by linking it with the past; otherwise risks might materialize in catastrophic consequences.

In particular, motivated by the need of mitigating future risks in power systems, this work focuses on finding robust and flexible investment strategies in the generation capacity expansion planning problem under exposure to multiple uncertainties. They are present in different sources and types such as fuel costs, investment and operational costs, demand growth, renewables variability, transmission capacity, environmental policies, and regulation. The problem when considering multiple uncertainties is much harder, not only because the increased computational effort, but also because it is hard to model the combination of their occurrences in a single optimization problem.

Since each uncertainty deserves special treatment, they are grouped into two categories. Those (categorical) uncertainties that really impact the portfolio investment decisions are classified as global; whereas those that quantitatively describe the intrinsic imperfect knowledge of the categorical are considered local uncertainties. So, to effectively account for robustness, defined as the ability to perform well under unforeseen situations, and flexibility, defined as the ability to adapt cost-efficiently to different situations, modern tools are illustrated and implemented in a computationally tractable manner, resulting in promising planning tools under uncertainty.

CHAPTER 1. OVERVIEW

1.1 Motivation

At any point in time, the world is changing and so are all of its components. Any society, country, or productive process continuously searches growth to meet special needs and requirements. Growth can be measured with economic indicators, or can be observed in terms of bigger infrastructure, or for some it can be perceived through increased consumption habits. The successful evolution of these systems is achieved when humans attempt to visualize what their future growth path will look like. However, in order to better visualize and quite interpret the future, it is crucial to understand there are situations and challenges that interact with every system, and that somehow they need to be overcome and thus avoid undesired catastrophic future situations.

Many of these situations and challenges come from the uncertain environment where every system performs. In particular, the power sector is one of those systems that needs special attention regarding both its evolution path and potential risks it might face. Currently, electricity has become more important and is a commodity every single person and sector is more dependent on. Electricity uses span from charging a battery of a personal computer to electrifying the transportation system. This wide spectrum of power demands create beneficial interdependencies between the power system with the rest of the world; but, at the same time the risks faced by "the rest of the world" are also transferred to the power sector.

Risk, understood as a situation exposed to potential danger, is present as long as more things remain unknown or are out of control. Unfortunately, apart from the future, there is plenty of incomplete knowledge and/or randomness regarding the forces that drive the economy, consumption patterns, politics, among many others (resource availability, weather). And

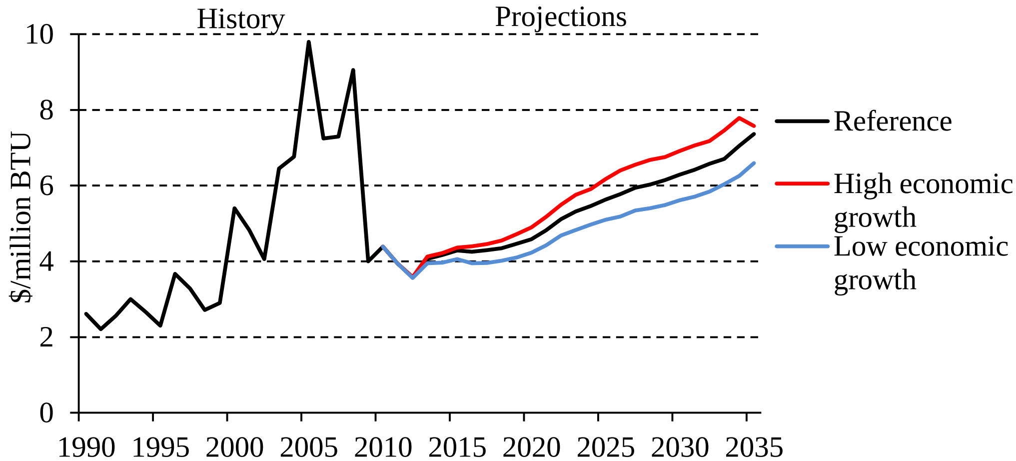


Figure 1.1: Natural gas price AEO

the current power sector, is exposed to these risks via its interdependencies with these other systems.

In particular, power demand, fuel prices, penetration of new technology, investment costs, market rules, future of fossil fuel generation technologies, power demand of transportation sector, renewable resources, and environmental isssues are some uncertainties to which the current US power system is exposed. For instance, Fig. 1.1, taken from Conti et al. (2012), displays the past and possible future trends in natural gas price depending on the US economic growth. Natural gas price has notably reduced in the last two years compared to what it was in 2005 for example. This has motivated the power sector to think of an electricity portfolio heavily composed of natural gas based power plants. Power demand has been growing continuously but at lower rates. According to the Anual Energy Outlook (AEO) 2012 of the Energy Information Administration (EIA), in the last decade, it has grown only at 0.7%. Increment of sales of both plug-in hybrid and electric cars, and user travel patterns altogether, have the potential to increase the power demand.

Policy has also impacted the state of the power sector. In the US, Federal and State regulations have been incorporated recently. For instance, the Cross-State Air Pollution Rule is a cap-and-trade system for SO_2 and NO_x emissions; the California Assembly Bill 32 is a cap-and-trade system for reducing greenhouse gas (GHG) emissions from 2013 to 2020; the Renewable Energy Production Tax Credit is an incentive that allows an income (tax credit) per unit of electric energy produced by renewable resources; and the Renewable Portfolio Standards determines the minimum levels of electricity to be produced by renewables in 30 states.

The current power system capabilities are not well suited to satisfy all requirements of the future. The requirements can be summarized as continuous satisfaction of increasing demand, cleaner power, low retail electricity prices, and resilient operation in the face of unforeseen events. These requirements indicate that a continuous planning must be performed to keep track of economic, societal, and political conditions. In Conti et al. (2012), for example, EIA has projected that most of the capacity additions between 2011 and 2025 will come mainly from natural gas given the high construction costs of other technologies and the uncertainty in GHG emission policies; and that the rest of the capacity additions will come from renewables and clean-coal units.

In general, the objectives of a planning task are to reduce the risks of uncertainties and to gain some control over the future by linking it with the past. A successful plan should assess what the risks are if the resulting decisions are implemented, and it should also predefine alternative strategies in case conditions change dramatically. The success in performance and growth of the power sector depends on how the system is planned under uncertainty. However, given the multiple sources of uncertainty, it is difficult to develop a successful plan which accounts for those uncertainties effectively.

The effect on the system of each uncertain situation or scenario can lead the decision maker to extreme decisions. Fig. 1.2 illustrates that if today's system is taken, for instance, in the direction of scenario 2, the future system can be under significant risk if the realized scenario is actually pointing towards opposite directions like scenarios 5 or 6. It is important, from the decision maker's standpoint, to understand what the future system will look like under different circumstances, and his/her actual decisions must be the outcome of careful analysis and risk mitigation techniques. An usual assumption in most of uncertainty modeling applications is the complete knowledge of scenarios. When studying cases under different scenarios, there might be another hidden level of uncertainty. For instance, if one scenario considers low gas price, e.g., $4/MMBTU[1], and another considers high gas price, e.g., $8/MMBTU, there is an implicit

[1] 1 MMBTU = 1 million British thermal unit (BTU)

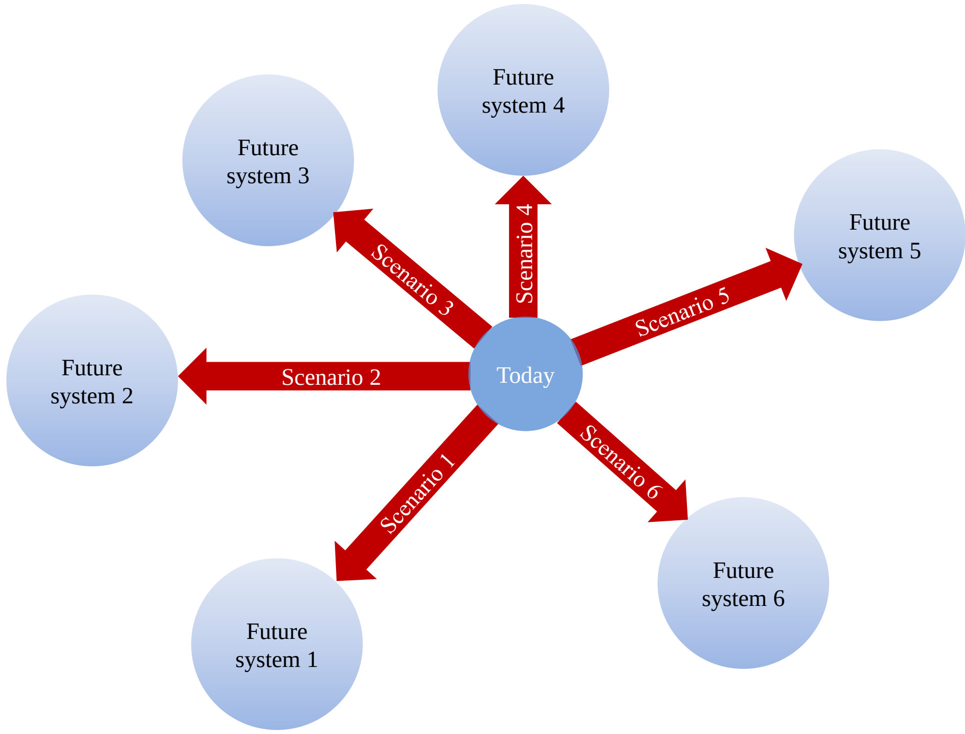


Figure 1.2: Effects of uncertainty

assumption of complete knowledge of both scenarios. But, is there any reason why a high gas price assumption could not be either \$7.5/MMBTU or \$8.9/MMBTU? Probably the answer is no. With the traditional mathematical tools it has been computationally expensive to address this issue; but, modern optimization tools can. This motivates the work of this dissertation in considering another level of uncertainty in scenario analysis.

The diverse uncertainty space faced by the power sector makes this decision-making problem hard to solve. Uncertainties should not be treated in the same way. Some can be modeled statistically by using historical information, e.g. fuel prices, power demand; and others have never occurred therefore there is no any information to characterize its behavior, e.g., regulation regarding renewables and/or GHG emissions. In addition, there is also a set of tools that have been employed to handle uncertainties; however, some of them have limitations regarding the number of uncertainties, or the dimensionality of the uncertainty space. The intention of this work is to develop new strategies that can be implementable for performing power system planning under multiple uncertainties.

1.1.1 Description of the problem

This work is commited to study the CEP problem under presence of different types and sources of uncertainty. In this context, the CEP problem consists of identifying the most cost-efficient energy portfolio balancing robustness and flexibility, i.e., to determining optimal investments in time and location of the best generation technologies that satisfy the future energy needs with minimum levels of risk, considering technical and environmental constraints, and multiple sources and sizes of uncertainty.

1.2 Objectives

The purposes of this work are to study the effects of uncertainties in the generation capacity expansion planning problem, and to identify and design the more suitable methodologies for uncertainty modeling in long–term planning. Suitable methodologies are those that: are not very sensitive to the assumptions made by the decision maker; can handle multiple sources of uncertainty and mitigate multiple kinds of risk; and are computationally tractable. In particular, the specific goals are to:

1. Provide a classification of uncertainties observed in the CEP according to their impact on the power system;
2. Develop expansion planning techniques capable of providing results that are economically feasible and robust to multiple sources of uncertainty data;
3. Develop expansion planning techniques capable of providing results that are economically feasible and flexible to multiple sources of high-impact uncertainties;
4. Improve the computational performance of the resulting models by the implementation of multi-stage decomposition methodologies.

The entire work is built using a capacity expansion model that provides the investment decisions needed to satisfy future energy needs and operational constraints modeled as a direct current optimal power flow (DCOPF). Since most of this work deals with a 40-year planning horizon, it would be computationally intensive to consider the operation of the system hour by hour. As an approximation, the operation of the system is considered for three different periods per year using the so-called load duration curve (LDC). To the DCOPF, we have provided features that add realism to the solution like maximum capacity factor and capacity credit. The first limits the the energy produced by each technology in a year; and the second limits the production of renewables like wind and solar according to the time of the day.

Throughout this work, multiple sources, types and amounts of uncertainties have been used. Investment cost, fuel prices, demand, capacity factor, capacity factor, transmission capacity, and environmental regulation, are some of the uncertainties considered. However, to properly model them, uncertainties are classified according to the impact on the results of running the planning tool. For instance, when changes in some data or parameter produce a significant different trend in the portfolio, that parameter is called a *global* uncertainty. Examples of global uncertainties are the implementation of emissions policies, important shifts in demand, unavailability of a resource such as coal or natural gas, regulation regarding nuclear plants operation, an important drop in investment costs, among others. To model the imperfect knowledge of the global parameter, *local* uncertainties are used and parameterized through uncertainty sets. In statistical terms, a local uncertainty is similar to a dependent random variable.

Each uncertainty type deserves special attention. In the case of local uncertainties, whose representation is valid via uncertainty sets, RO is a suitable tool. Under RO, the CEP can be impacted by different sources and sizes of uncertainty. However, when the CEP results are very sensitive to some uncertainty, RO is not appropriate. Although robustness in a solution is crucial, it can be too expensive. That is the reason why under global uncertainties, the concept of flexibility in planning is much more useful and practical. Basically, it is a criterion imposed to the CEP that ensures the final solution can be continuously adapted to the conditions of different scenarios at minimum cost. However, there is a tradeoff between investment and adaptation cost that needs to be considered: costly portfolios are quite robust and need little adaptation to other scenarios; conversely, low-cost portfolios are not robust and incurr in high costs in adapting to other scenarios.

Each of the planning solutions are tested for robustness via Monte Carlo simulation. It is interesting how, without extreme differences in the solutions, the uncertainty-based solutions always outperform the deterministic ones in terms of risk at little additional investment cost.

The kind of tools presented in this work are computationally applicable to real power systems because they can handle multiple uncertainties. This feature is not achieved even by traditional tools like Stochastic Programming (SP). Also, constructing uncertainty sets only requires the bounds of the uncertain parameters, compared to SP which requires processing more information to obtain probability distributions. In cases where uncertainties are not parameterizable by these sets, modeling flexibility in planning is a promising new concept in that portfolios can adapt to different scenarios at minimum cost. Decision makers, investors, government agencies can take adavantage of these types of methodologies to analyze the effect of new policies and the risk exposure of the system. Also, Independent System Operators can use these methodologies to assess the risks in a more rigorous way. Furthermore, optimization based planning tools under uncertainty are useful for providing signals regarding the fields in which technology development and research efforts need to be made.

1.3 Thesis organization

This chapter presents the motivating aspects of doing this work and its objectives. Chapter 2 is a literature review of the traditional and modern tools used in decision–making problems under uncertainty. Chapter 3 presents a paper that explains some concepts of RO and its implementation in the planning of power system. Chapter 4 presents a RO model applied to the CEP that uses affine decision rules as function of uncertainties for decreasing the conservatism level of the solution. Chapter 5 shows how the model proposed in Chapter 4 is transformed such that it can be solved alternatively by a decomposition method called dual dynamic programming. Chapter 6 presents a methodology that involves the modeling of global and local uncertainties jointly through the concept of flexibility. Finally, chapter 7 discusses the major findings of this work and provides research directions.

CHAPTER 2. REVIEW OF LITERATURE

For over 50 years, researchers have been thinking about solving optimization problems under presence of uncertainty; as a result, a diverse world of methods and philosophies have been studied. This chapter summarizes the basic elements of the methods that will be more prevalent in this work and the most significative efforts in the field, emphasizing those used in power system applications.

2.1 Sensitivity Analysis

Traditionally, Sensitivity Analysis (SA), has been an essential approach for identifying the influence on simulation caused by changes in input data. Most research fields utilize SA to better understand results. In the field of optimization in particular, it can be seen as a post-optimization tool. A SA does not alter the problem structure, it only provides different parameters to the actual optimization problem. This approach does not provide a solution that is protected against unforeseen uncertainties in general. Yet, SA does provide a preliminary understanding of the effects, which is valuable. It is possible that some people make decisions combining different results obtained by SA. In conclusion, nothing but sensitivities can be obtained using a SA method simply because that is its unique purpose.

2.2 Stochastic Programming

SP has been widely used as powerful tool that does include an uncertainty model into the mathematical formulation of the problem. Basically, by making use of probability distributions of uncertain data, an stochastic program considers the minimization of the expected costs as explained in Shapiro et al. (2009), Birge and Louveaux (2009). In some applications, it also considers the minimization of a risk measure as in Malcolm and Zenios (1994). For multi-stage problems, it requires the structure of a scenario tree by approximating each random variable with a fixed number of samples. Besides uncertainty approximation, the most critical disadvantage that impedes development of realistic applications is the exponential growth of the number of scenarios with the number of time steps, making the problem computationally intractable in general (Ben-Tal and Nemirovski (1998), Ben-Tal et al. (2009)).

Under perfect foresight, the objective function has a defined value under a decision; but, under random data, there are several outcomes. In the SP setup, a decision maker prefers to optimize the average cost. Sometimes, decision makers face the problem of making a "safe" decision no matter what the outcomes are. This constitutes a risk-averse attitude and results in high-cost decisions. To reduce costs, decision makers can accept small levels of risk in their decisions by properly penalizing the recourse variables.

One of the ways to address these issues is by using two-stage stochastic programs. The decision maker needs to make here-and-now decisions (which cannot wait until data is revealed) and, wait-and-see decisions (those that are implemented once data is observed). In the context of power systems, here-and-now decisions are the generation levels for each unit in the grid resulting from the economic dispatch problem. These levels are scheduled before actual demand is observed. And wait-and-see variables are, for instance, voltage angles, which are the result of a particular demand value. Within an expansion planning problem, here-and-now decisions are the $t = 0$ investment decisions. These need to be implemented before the multiple uncertainties are observed. The rest of variables that complete the description of a planning model, e.g., the $t > 0$ investment decisions, are wait-and-see variables.

Mathematically, a general two-stage program model is (Shapiro et al. (2009))

$$\begin{aligned} \underset{x \in \Re^n}{\text{minimize}} \quad & c^\top x + \mathbb{E}\left[Q\left(x, \xi\right)\right] \\ \text{subject to} \quad & Ax \leq b \end{aligned}$$

where x are the here-and-now variables, and the function $Q(x, \xi)$ is the optimal value of the second stage program. ξ represents the randomness with known distribution.

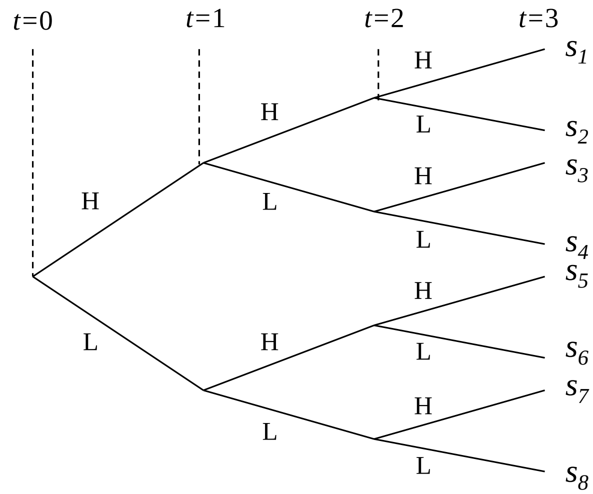


Figure 2.1: Scenario trees in stochastic programming

$$\begin{aligned} \underset{y(\xi)\in\Re^m}{\text{minimize}} \quad & q(\xi)^\top y(\xi) \\ \text{subject to} \quad & T(\xi)\, x + W\, y(\xi) \le h(\xi) \end{aligned} \tag{2.1}$$

Model (2.1) represents the model of the second stage for a specific data realization ξ. Its decision vector y is called recourse. The set of inequalities $Tx \le h$ represent the constraints of the second stage; however, if decisions x do not satisfy them, the recourse variables compensate the inconsistency through Wy. Therefore, $q^\top y$ is the cost of recourse.

In two-stage power system planning, assuming uncertain demand, x represents the capacity investment decisions that need to be made before demand is revealed. The recourse variables represent all the optimal power flow (OPF) decision variables under each demand scenario. However, since there exists the risk that demand cannot be met in the actual operation with the x decisions, another set of variables composed of demand deficits is also part of the recourse vector y. Thus, q represents the cost of both the energy deficit and operation.

Usually, uncertain data are sampled to create scenarios. In this case, the model (2.1) is replicated as many times as scenarios. This is an issue in the general multi-period case where the number of combinations of uncertainty realizations increases exponentially in time. For instance, Fig. 2.1 shows that the observation of a binary random variable with values H and L during 3 periods yields to $2^3 = 8$ total scenarios. Also, each scenario needs a representation of the system under study which in turn will also increase dramatically the problem size. Realistic applications would have astronomical number of scenarios. To handle this, scenario sampling techniques have been used. Refer to the books of Kall and Wallace (1994); Prékopa (1995); Birge and Louveaux (2009); Shapiro et al. (2009) for deeper understanding of stochastic programming.

2.3 Robust Optimization

Besides stochastic programming, RO has emerged as a promising research area in operations research literature like Ben-Tal and Nemirovski (1998), Ben-Tal et al. (2009), Ben-Tal and Nemirovski (1999), Ben-Tal and Nemirovski (2000), Ben-Tal and Nemirovski (2002), Bertsimas and Sim (2003), Bertsimas and Sim (2004), Sim (2004), Bertsimas et al. (2011a). References Ben-Tal et al. (2009) and Ben-Tal and Nemirovski (2002) discuss the potential RO has of being applicable in many disciplines. The work in Bertsimas et al. (2011a) mentions that robust optimization is being used in antenna design, integrated circuit design, network flows and traffic management, wireless networks, robust control, model adaptive control, portfolio management, inventory control, statistics and parameter estimation. Bertsimas and Sim (2003) show the mathematical formulation for combinatorial optimization and network flow problems, and Alem and Morabito (2012) present an application of RO in production planning considering demand and cost uncertainties. The work in Verderame and Floudas (2011) is an operational planning of a multi-site production and distribution network considering demand and transportation time uncertainty. The work of Soyster (1973) was the first attempt to consider a linear program with box–shaped uncertainty in the parameters of the linear constraint. However, it was up to the late 90's that the area became popular with the works of Ben-Tal and Nemirovski.

Unlike SP, RO avoids the representation of scenario trees and the sampling process; rather, it assumes that the uncertainty space of data is constrained to an uncertainty set and finds the best solution that is feasible for all the realizations of uncertainties that lie in the uncertainty space under consideration. Fig. 2.2 shows how an ellipsoidal uncertainty set is used to approximate uncertain data. However, neither is this ellipsoidal shape a requirement nor many data points are needed to create the sets. When data points are few, one can create a box-shaped

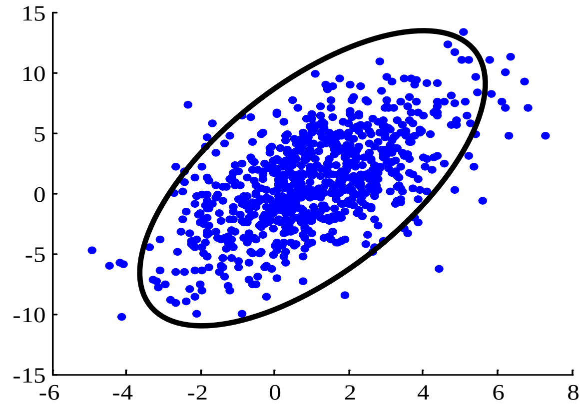


Figure 2.2: From data to uncertainty sets

set to be "safe."

Increasing the "safety" or robustness will definitely worsen the objective function, this is what Bertsimas and Sim (2004) call the price of robustness. This price is higher as long as the solution becomes more conservative (robust); however, the level of conservatism can be controlled according to the risk preferences of the decision-maker. The work in Bertsimas and Sim (2004) also presents the RC of cardinality constrained uncertainty where the conservatism level, defined by their uncertainty budget, is controlled by the number of uncertain parameters that actually vary from their nominal values. This type of formulation is promising when dealing with contingencies in security-related applications.

In this section, we show how an optimization problem with uncertain data characterized by uncertainty sets is transformed to its RC. In order to explain RO and its underlying guidelines, we use the main concepts adapted from Ben-Tal et al. (2009).

Consider the following uncertain linear program:

$$\begin{aligned} \underset{x \in \Re^n}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & a_i^\top x \leq b_i,\ i = 1, \ldots, m. \\ & (A, b, c) \in \mathcal{U} = \mathcal{U}^A \times \mathcal{U}^b \times \mathcal{U}^c \end{aligned} \tag{2.2}$$

$A \in \Re^{m \times n}$, $b \in \Re^m$, and $c \in \Re^n$ are arrays of uncertain parameters that lie in a convex uncertainty set $\mathcal{U}$ defined on $\Re^{m \times n} \times \Re^m \times \Re^n$ as the cartesian product of each uncertain parameter uncertainty set. It is assumed that each $a \in [\bar{a} - \hat{a}, \bar{a} + \hat{a}]$, $b \in [\bar{b} - \hat{b}, \bar{b} + \hat{b}]$, and $c \in [\bar{c} - \hat{c}, \bar{c} + \hat{c}]$, where $\hat{a}$, $\hat{b}$ and $\hat{c}$ are the maximum variations of a, b, and c with respect to their nominal values $\bar{a}$, $\bar{b}$, and $\bar{c}$, respectively.

The RO approach deals with finding a solution to the linear program (2.2) such that it is feasible under any realization of the uncertain parameters. When the parameters are not only considered uncertain but also random, i.e. they have a probability distribution, the RO formulation is still applicable. In fact, references Bertsimas and Sim (2004), and Ben-Tal et al. (2009) use a probability indicator to measure the level of satisfaction of the constraint. In this case, RO and chance-constrained optimization become highly related (see Section 2.5).

From here, without loss of generality, we will consider the linear program (2.2) with only one constraint of the form $a^\top x - b \leq 0$. In general, a RO problem is solved by solving the following model:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \sup_{c \in \mathcal{U}^c} c^\top x \\ \text{subject to} \quad & \sup_{(a,b) \in \mathcal{U}^a \times \mathcal{U}^b} \left(a^\top x - b \right) \leq 0 \end{aligned} \tag{2.3}$$

This formulation ensures that under *any* observation of uncertainty within the bounds defined by $\mathcal{U}$, the solution will be feasible. Furthermore, the minimization of the maximum value of the objective function, known as the worst-case scenario, is implemented; therefore, the optimal objective function is indeed an upper bound of the actual objective value given the uncertainty represented in c.

Without loss of generality, we are only considering uncertainty in a and b. By using affine perturbations $\eta \in \mathcal{Z}$ where $a_k = \bar{a}_k + \eta_k \hat{a}_k,\ k = 1, \ldots, n$, and $b = \bar{b} + \eta_{n+1} \hat{b}$, the linear program (2.3) can be written as:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^{n} \bar{a}_k x_k - \bar{b} + \sup_{\eta \in \mathcal{Z}} \left(\sum_{k=1}^{n} \eta_k \hat{a}_k x_k - \eta_{n+1} \widehat{b} \right) \leq 0 \end{aligned} \tag{2.4}$$

It is important to appropriately choose the primitive uncertainty set $\mathcal{Z}$. This selection will yield a different structure of the problem (2.4).

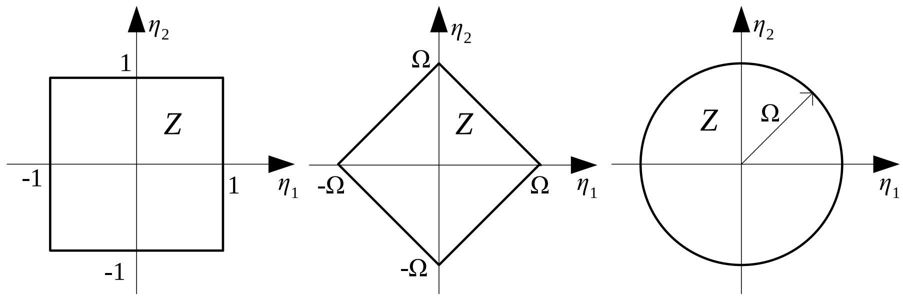


Figure 2.3: Usual uncertainty Sets

In each case Ω is known as the uncertainty budget. This parameter is used to tune the amount of uncertainty modeled in the problem. This parameter is necessary when $\hat{a}, \hat{b}$, and $\hat{c}$ are only an approximation of the data bounds, and therefore, risk level has to be controlled. For the case $\Omega = 1$, the uncertainty set bounds are exactly the data bounds. So, the larger the Ω, the more uncertainty modeled; therefore, the less risky the solution will be and vice versa. The intuition here is if the "size" (area, volume, etc.) of the uncertainty is large —usually $\Omega > 3$, the decision vector x will be more protected against dangerous realizations of data. Figure 2.3 graphically shows the shape of these three types of uncertainty sets $\mathcal{Z}$ when the uncertainty is two-dimensional.

Now, in order to find a tractable way of solving the problem (2.4), it is necessary to find an analytical solution (if any) of the optimization subproblem with decision variables η_1 to η_{n+1}. Of course, for every selection of $\mathcal{Z}$, there will exist a solution, and the equivalent deterministic optimization problem (2.4) or RC will have a different structure.

2.3.1 Box Uncertainty

The primitive uncertainty set is defined as

$$\begin{aligned}\mathcal{Z} &= \left\{\eta \in \Re^{n+1}, \|\eta\|_\infty \leq 1\right\}\\ &= \left\{\eta \in \Re^{n+1}, |\eta_k| \leq 1,\ k = 1, \ldots, n+1\right\}\end{aligned}$$

The optimization of the second-level problem in (2.4) with respect to the disturbance vector

η is a linear program, and its solution is

$$\eta_k^* = \frac{|x_k|}{x_k} = \begin{cases} 1 & , \text{ if } x_k > 0, \\ 0 & , \text{ if } x_k = 0, \\ -1 & , \text{ if } x_k < 0, \end{cases}$$

and $\eta_{n+1}^* = -1$.

Therefore, the robust counterpart of (2.4) is

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^{n} \bar{a}_k x_k + \sum_{k=1}^{n} \hat{a}_k |x_k| + \hat{b} \leq \bar{b} \end{aligned}$$

or equivalently

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \bar{a}^\top x + \Omega \left\| \left[\hat{A}\, x;\ \hat{b} \right] \right\|_1 \leq \bar{b} \end{aligned} \tag{2.5}$$

$\hat{A}$ is a diagonal matrix of $\hat{a}_k$ elements.

However, in order to maintain feasibility and/or robustness, optimality of the solution can be lost at a significant level. The decision maker would end up paying too much for protecting its solution against any uncertainty realization in the box. Indeed, this is considered the most conservative or risk averse uncertainty set in the sense that the model considers the worst case possible values of data. In applications involving large amounts of data, it is not straightforward to determine the the worst-case combinations of data. Under the box uncertainty set, it is not necessary to explicitly specify the worst-case scenario, model (2.5) takes care of that.

2.3.2 Ellipsoidal Uncertainty

The primitive uncertainty set is written as

$$\begin{aligned} \mathcal{Z} &= \left\{ \eta \in \Re^{n+1}, \|\eta\|_2 \leq \Omega \right\} \\ &= \left\{ \eta \in \Re^{n+1}, \sqrt{\eta_1^2 + \eta_2^2 + \ldots + \eta_{n+1}^2} \leq \Omega \right\} \end{aligned}$$

The optimization of the second-level problem in (2.4) with respect to the uncertain vector η belonging to an ellipsoidal uncertainty is a simple convex optimization problem. It has an analytical solution given by

$$\eta_k^* = \frac{\hat{a}_k x_k}{\sqrt{\sum_{j=1}^n \hat{a}_j^2 x_j^2 + \hat{b}^2}}\Omega,\ k = 1, \ldots, n$$
$$\eta_{n+1}^* = -\frac{\hat{b}}{\sqrt{\sum_{j=1}^n \hat{a}_j^2 x_j^2 + \hat{b}^2}}\Omega$$

Therefore, the robust counterpart of (2.4) is

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^n \bar{a}_k x_k + \Omega \sqrt{\sum_{k=1}^n \hat{a}_k^2 x_k^2 + \hat{b}^2} \leq \bar{b} \end{aligned}$$

or equivalently

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \bar{a}^\top x + \Omega \left\| \left[\hat{A}\, x;\ \hat{b} \right] \right\|_2 \leq \bar{b} \end{aligned} \tag{2.6}$$

2.3.3 Manhattan Uncertainty

The primitive uncertainty set is represented as

$$\begin{aligned} \mathcal{Z} &= \left\{ \eta \in \Re^{n+1}, \|\eta\|_1 \leq \Omega \right\} \\ &= \left\{ \eta \in \Re^{n+1}, |\eta_1| + |\eta_2| + \ldots + |\eta_{n+1}| \leq \Omega \right\} \end{aligned}$$

The optimization of the second-level problem in (2.4) with respect to the uncertain vector η is a linear program, and its solution is therefore an extreme point of the uncertainty set:

$$\eta_k^* = \begin{cases} \Omega & , \text{if } k = j, \\ 0 & , \text{if } k \neq j \end{cases}$$

and the j-th component is chosen such that

$$t_j = \max\left(\hat{a}_1 |x_1|, \ldots, \hat{a}_n |x_n|, \hat{b} \right)$$

Then, the RC of (2.4) becomes

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^{n} \bar{a}_k x_k + \Omega \max\left(\hat{a}_1 |x_1|, \ldots, \hat{a}_n |x_n|, \hat{b}\right) \leq \bar{b} \end{aligned}$$

or equivalently

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c\top x \\ \text{subject to} \quad & \bar{a}^\top x + \Omega \left\| \left[\hat{A}\, x;\ \hat{b}\right] \right\|_\infty \leq \bar{b} \end{aligned} \tag{2.7}$$

The resulting RCs correspond to different types of optimization problems. When using the box and Manhattan uncertainty sets, the RCs (2.5) and (2.7) are still linear programs; if slight modifications (including auxiliary variables and constraints) are made, that can be seen more clearly. But, the ball uncertainty yields a convex nonlinear program (2.6), more specifically, a second order cone program. More remarkable, each of these RCs are computationally tractable.

2.3.4 Polyhedral uncertainty

In Bertsimas et al. (2011a), the RC is developed for a general polyhedral uncertainty set as follows:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \max_{D_i a_i \leq d_i} a_i^\top x \leq b_i,\ \forall i = 1, \ldots, m \end{aligned} \tag{2.8}$$

Assume uncertainty comes from vectors a_i, $\forall i = 1, \ldots, m$. The matrix D_i define the polyhedron of uncertain data involved in the i-th constraint. To obtain the RC, duality arguments are used. The dual problem of the second-level optimization corresponding to the i-th constraint

$$\begin{aligned} \underset{a_i}{\text{maximize}} \quad & a_i^\top x \\ \text{subject to} \quad & D_i a_i \leq d_i : \pi_i \end{aligned}$$

is given by

$$\begin{aligned} \underset{\pi_i}{\text{maximize}} \quad & d_i^\top \pi_i \\ \text{subject to} \quad & D_i^\top \pi_i = x \\ & \pi_i \geq 0 \end{aligned}$$

Then, the RC of (2.8) becomes

$$\begin{aligned} \underset{x,\,\pi}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & d_i^\top \pi_i \leq b_i,\ \forall i = 1, \ldots, m \\ & D_i^\top \pi_i = x,\ \forall i = 1, \ldots, m \\ & \pi_i \geq 0,\ \forall i = 1, \ldots, m \end{aligned} \tag{2.9}$$

This formulation holds even for the cases when data vector a is random but bounded by a polyhedral uncertainty set.

2.3.5 Random bounded data

Let $\bar{a} = \mathbb{E}\, a$ and $\Sigma = \mathbb{E}\,(a - \bar{a})\,(a - \bar{a})^\top$. $\bar{a}$ and Σ are the expected value and the covariance matrix of the random vector a respectively. If η is assumed to be an uncorrelated random vector with $\mathbb{E}\,\eta = 0$ and $\mathbb{E}\,\eta\,\eta^\top = I$, a can be expressed as

$$a = \bar{a} + \Sigma^{1/2}\eta,\ \ \eta \sim (0, I)$$

The robust counterpart becomes

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \max_{D_i a_i \leq d_i, a_i \sim (\bar{a}, \Sigma)} a_i^\top x \leq b_i,\ \forall i = 1, \ldots, m \end{aligned} \tag{2.10}$$

The RO problem (2.10) can be posed in terms of the disturbance vectors η_i as follows:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & a_i^\top x + \max_{D_i \Sigma_i^{1/2} \eta_i \leq d_i - D_i \bar{a}_i} \eta_i^\top \Sigma^{1/2} x \leq b_i,\ \forall i = 1, \ldots, m \end{aligned}$$

And applying the results of Bertsimas et al. (2011a) regarding polyhedral uncertainty sets, the RC of (2.10) becomes

$$\begin{aligned} \underset{x,\,\pi}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & a_i^\top x + (d_i - D_i \bar{a}_i)^\top \pi_i \le b_i,\ \forall i = 1,\ldots,m \\ & \left(D_i \Sigma_i^{1/2}\right)^\top \pi_i = \Sigma_i^{1/2} x,\ \forall i = 1,\ldots,m \\ & \pi_i \ge 0,\ \forall i = 1,\ldots,m \end{aligned}$$

which is exactly (2.9) because the equality becomes $D_i^\top \pi_i = x$ given the symmetry of $\Sigma^{1/2}$.

Thus, results in (2.9) hold for random data under *any* probability distribution with any second moments. Based on this analysis, what actually matters is the geometric shape of the uncertainty set. The domain of the uncertainties is what matters for the model rather than the actual support of the distribution.

When the decision maker is well informed and has a good representation of the uncertainties in terms of probability distributions, chance-constrained optimization models can be more useful as will be discussed in Section 2.5.

2.4 Adjustable Robust Optimization

A RO solution has a significant conservatism level in order to maintain feasibility; and in multistage optimization, a robust solution might be even more conservative since all decisions (for all periods) are made at time zero to guarantee present and future feasibility. It would be useful for a delay in time before analyzing some revealed information to improve the decisions. This idea helps to avoid extra-conservatism and improve flexibility in the optimization. To develop this idea, like two-stage SP, the actual decisions, i.e., here and now and wait and see decision variables have to be differentiated. An approach that develops this methodology combined with a robust formulation of the problem is the so-called ARO in Ben-Tal et al. (2009) and Ben-Tal et al. (2004), or adaptable robust optimization in Caramanis (2006). The works presented by Chen et al. (2007) and Chen et al. (2008) show applications of ARO in SP.

The way decisions are made in ARO is by arbitrarily constructing decision rules that are function of past or revealed information. This process allows decisions to adjust according previous realization of uncertainties, which is similar to the idea of nonanticipativity conditions in stochastic programming. Decision rules are set only for the wait-and-see variables such that they can take corrective actions to improve the bad situations that could have happened during previous stages.

A common approximation for setting the ARO is using affine decision rules, i.e., future investment decisions (investments in power system planning) are parameterized as affine functions of observed data. Thus, the resulting optimization problem, which is still linear under some assumptions, results in the AARC (Ben-Tal et al. (2009), Ben-Tal et al. (2004)). Among the advantages of using the AARC is the adaptability of the solutions, the reduction of the objective function (cost minimization), computational tractability, and robustness of the solution. This type of decision rules are also called linear decision rules (Chen et al. (2008)).

Adaptability refers to solutions that can self-adjust according to the optimal decision rule. Deviations in data below or above the expectations during some stage(s) are inputs in the computation of future decisions. For example, if natural gas price exceeds its price expectations, it might be more cost-efficient to have less natural gas power capacity and more capacity coming from other resources. A natural gas investment decision rule might teach us the same rationale by using the AARC and making natural gas price part of the uncertainty affine rule. Adaptability avoids making future immediate expensive decisions caused by unexpected disturbances in data.

Apart from adaptability, the objective function is reduced. The increased number of degrees of freedom in the ARO helps to mitigate even more the variability in constraints and objective function, which in turn can achieve a lower cost solution compared to the RO solution (see Chapters 4 and 5). Although the approach considers recourse, the resulting optimization is still tractable for finite horizon problems. Finally, robustness is another goodness of this approach. Although solutions are not known so far in advance because we have to wait until uncertainties are observed, the feasibility of the solutions is guaranteed since the underlying modeling technique is RO.

The work Ben-Tal et al. (2004) is one of the first that introduced the idea of adjustable solutions in optimization. Since then, several theoretical improvements and practical applications have been reported. Caramanis (2006) presents a comprehensive development of what authors call adaptable RO and illustrates an application of the approach on air traffic control under weather uncertainty. Ben-Tal et al. (2005) use an AARC to study a multistage supply chain problem under uncertainty in demand by minimization of the worst-case cost function. Adida and Perakis (2010) present some models that incorporate uncertainty in a dynamic pricing and inventory control problem, their AARC approach outperforms dynamic programming, static RO, and stochastic programming. In the literature of power systems, no ARO-related works have been officially reported. The work by Bertsimas et al. (2011b) has not been published yet. It shows an application of adaptable robust optimization in unit commitment with load uncertainty.

2.5 Chance-constrained optimization

When constraints are contaminated with random uncertainties in data and some of the constraints can be relaxed to some degree, a chance-constrained optimization model is useful. Here, the decision maker can arbitrarily choose an admissible small probability of violating the constraint. So, his/her solution will be such that under most realizations of uncertainty, it is feasible.

Among the first contributions in the field of chance constrained optimization are references Charnes and Cooper (1959), Miller and Wagner (1965), Prékopa (1970), and Prékopa (1995). Since chance-constrained optimization and robust optimization are related, Ben-Tal et al. (2009) show safe approximations of chance constrained optimization via robust optimization. A chance-constrained program is convex and tractable in some cases; if data are jointly normally distributed and the admissible violation probability is less than 1/2, the problem can be converted to a second-order cone programming problem and therefore is convex (Boyd and Vandenberghe (2004)).

Chance-constrained and RO have a strong relationship. In RO, data do not have to be random and the solution is worst-case oriented based on the uncertainty set. In chance-constrained optimization, data are random and the solution is not necessarily feasible under any data realization. Rather, constraints are allowed to be violated with a small probability ϵ. In other words, rather than having the usual constraint $a^\top x \le b$, the constraint is $P(a^\top x \le b) \ge 1-\epsilon$ is used.

In general, chance-constrained optimization problems are intractable (Ben-Tal et al. (2009)). This is because, the inequality $a^\top x - b \le 0$ involves the sum of n random variables, which implies n-dimensional integration for computing the probability. And second, the set under which $P(a^\top x \le b) \ge 1-\epsilon$ is nonconvex in many cases. However, only a few cases have an exact deterministic safe representation:

- b is random: $b = \bar{b} + \sigma_b \omega,\ \omega \sim (0,1)$

  ω is any random variable with mean zero and variance one with cumulative probability distribution F_ω. The equivalent deterministic constraint is

$$P(a^\top x \ge b) \le \epsilon \Leftrightarrow a^\top x \le \bar{b} + \sigma_b F_\omega^{-1}(\epsilon)$$

- a multivariate normal: $a \sim N(\bar{a}, \Sigma)$

  The tractability in this case comes from the fact that the quantity $\bar{a}^\top x$ is a normal random variable. Therefore, it is easy to write the deterministic constraint in terms of the cumulative distribution of the standard normal Φ:

$$\bar{a}^\top x + \Phi^{-1}(1-\epsilon) \left\|\Sigma^{1/2} x\right\|_2 \le b$$

  This constraint describes a convex set as long as $\epsilon < 1/2$.

- a is any distribution: $a \sim (\bar{a}, \Sigma)$

  In Ben-Tal et al. (2009) it is shown that for a random vector a under *any* distribution

$$P_a\left(a^\top x > b\right) \le \exp\left\{-\frac{\left(b - \bar{a}^\top x\right)^2}{2\left\|\Sigma^{1/2} x\right\|_2^2}\right\}$$

  then, if the constraint $a^\top x \le b$ is allowed to be violated with probability ϵ, then the condition

$$\epsilon \le \exp\left\{-\frac{\left(b - \bar{a}^\top x\right)^2}{2\left\|\Sigma^{1/2} x\right\|_2^2}\right\}$$

has to hold under the decision vector x. So, the approximate chance constraint can be written as

$$\bar{a}^\top x + \sqrt{2\ln(1/\epsilon)} \left\| \Sigma^{1/2} x \right\|_2 \leq b$$

In every case, the safe version of the constraint $P(a^\top x \leq b) \geq 1 - \epsilon$ is nothing but the mean $\bar{a}^\top x$ plust a safety term depending on ϵ times the standard deviation $\left\| \Sigma^{1/2} x \right\|_2$. The safety term is usually less than 3 for $\epsilon < 0.01$.

2.6 Decision theory

Probabilistic methods like stochastic programming are useful for handling random uncertainties (Buygi et al. (2006)). But, these methods based on distributional assumptions might be inappropriate in the case of nonrandom uncertainties where policies, preferences, and government decisions cannot be modeled in terms of distributions (Kouvelis and Yu (1997)).

However, in what there seems to be a consensus in the literature, handling nonrandom uncertainties can be done by decision analysis theory. The main concept is that decision makers need to look at the problem from a decision point of view rather than from an optimization point of view. Decision makers are forced to see what is the best that could have been done if he/she had known in advance the occurrence of a specific scenario, and his/her best solution would be the one that shows the most similar performance to the benchmark of the scenario Kouvelis and Yu (1997). That is, decision makers want to feel the least *regret* caused by not having made the best decision under a specific scenario. Or, as it has been recently developed in Zhao et al. (2009), decision makers have to really measure the consequences or the effort to adapt to new circumstances by not having made the best decision; therefore, they can select the best decision as the one requiring the least effort to adapt.

2.6.1 Regret minimization

Mathematically, the regret felt R when implementing decision x under scenario s is

$$R(x, s) = f(x, s) - f(x^*, s)$$

where $f(\cdot)$ denotes an attribute that has to be minimized (cost, deficit of resource) and x_s^* is the best solution under scenario s. If $R(x^r, s) = 0$, $\forall s$, then x^r is said to be robust. So, a risk averse decision maker wants to solve the following problem:

$$\min_x \max_s R(x, s)$$

The works by Gorenstin et al. (1993); Maghouli et al. (2011); Miranda and Proenca (1998a); De la Torre et al. (1999); and Fang and Hill (2003) use the minimax regret method in scenario-based planning problems for dealing with nonrandom uncertainties. Miranda and Proenca (1998a) explain why the probabilistic choice is an apriori evaluation, i.e. decisions are made before scenario occurs; whereas risk analysis tools (minimization of regrets) is a posteriori evaluation since it is based on consequences of scenario occurrence.

The regret of a solution within a scenario is defined as the difference between the social cost of the solution under the scenario and the social cost of the optimal solution of the scenario. A regret equal to zero implies the solution is completely "robust" under the scenario; otherwise, the solution fails under conditions of the scenario. Given this, regret minimization does not require use of distributions to evaluate the performance of the solutions in different scenarios. Miranda and Proenca (1998b) comment that probabilistic approaches are not as effective for dealing with uncertainties as a regret minimization tool (risk analysis tool). A probabilistic method, according to Miranda and Proenca (1998b), chooses the optimal solution based on the average of futures with some probabilities, and therefore is riskier (allows solutions with higher regret values for catastrophic futures); and is therefore not good from a decision-making point of view. The risk analysis tools avoid selecting a solution with a bad performance in any future considered. Linares (2002) explains the importance of managing risk in power systems in order to achieve robust strategies conditional on a set of scenarios. Strategies that combine the analysis of the solutions of individual scenarios in order to obtain the solution that best performs in all scenarios are explained in Linares (2002) and Firmo and Legey (2002). Optimizing by scenarios might yield to local responses, but it does help in understanding of the impact of individual uncertainties on the system Linares (2002). However, obtaining only one solution that covers all (or many) of the uncertainty sectors could be more important. This solution is

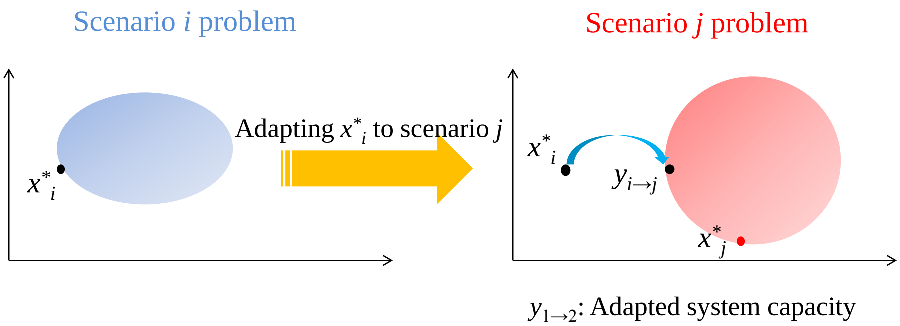


Figure 2.4: Adaptation problem

referred to as robust.

One of the disadvantages of this approach is its high sensitivity to the choice of scenarios as mentioned by Higle and Wallace (2002). The reason is that regrets evaluation is done among candidate solutions or decisions that are already proper of the scenarios under wich were obtained. Indeed, the same reference argues that this risk mitigation tool is too risky by itself.

2.6.2 Minimization of adaptation costs

A recent approach designed to deal with consequences of nonrandom uncertainties is based on the concept of flexibility Zhao et al. (2009) and Zhao et al. (2011). When an expansion plan is designed using either a probabilistic approach or regret minimization, once the uncertainties (global) are revealed, the plan may not satisfy the system requirements and needs further adjustment or re-expansion. The reason is the plan would take several years to be implemented, and therefore it is likely that the forecasting of the scenario trajectory results will be wrong. Basically, the plan has to be adapted in a timely and cost-effective way to the new conditions defined by the just-observed scenario. Based on this argument, Zhao et al. (2009) propose an indicator that takes into account how much it costs to adapt a plan to the conditions of an observed scenario.

Mathematically, adaptation cost $AC(x, s)$ is the cost of adapting the planning solution x to scenario s. It can be computed by re-running the optimization model with data corresponding to scenario s starting with the current infrastructure x. Fig. 2.4 is a graphical description of what adapting a planning solution x_1 to scenario 2 means. Adapting a system infrastructure refers to finding the investment decisions that guarantee the system will meet all of the requirements under scenario s. In the plot, $y_{1,2}$ is the new infrastructure, and the difference $y_{1,2} - x_1$ determines the re-investments for a succesful adaptation. Then, according to Zhao et al. (2009), once the decision maker knows all the possible adaptation costs between scenarios, he/she wants to select the most flexible planning solution by choosing the solution that is cheaper to adapt to the worst-case scenario:

$$\min_{x} \max_{s} AC(x, s)$$

The works of Zhao et al. (2009), Maghouli et al. (2011), and Zhao et al. (2011) appear to be the only applications that consider adaptation cost in the power system literature. Related concepts of flexibility and core solutions are presented in Balijepalli and Khaparde (2010).

2.7 Recent Uncertainty Approaches in Power System Applications

Many power system problems contain significant uncertainties, and researchers have dealt with them in different ways according to the prior knowledge of the uncertainty.

Stochastic programming

Sanghvi et al. (1982) present a generation expansion planning model that models uncertainty in load growth, load shape, unit availability, fuel availability, and weather conditions. Meza et al. (2007) consider fuel price uncertainties in a multi-objective power system expansion planning approach by minimizing costs and carbon emissions. The work by Gorenstin et al. (1993) is an application of SP in power system planning considering uncertainties in demand growth, fuel cost, delay in project completion, and financial constraints. The paper of Yehia et al. (1995) is an application to the Lebanese system that considers scenarios for demand increase rate, transmission planning and reactive power. Mo et al. (1991) minimize the expected investment and operation costs with Markov chains models for uncertainties and stochastic dynamic programming. Malcolm and Zenios (1994), in addition to expected cost, also consider second moment to minimize cost risk considering a discrete probability mass function for the scenarios. The work in Street et al. (2009) presents the selection of optimal renewable portfolios using stochastic programming considering spot price uncertainties risk and measured through conditional value at risk. Lopez et al. (2007) present a two-stage stochastic programming model for dealing with random uncertainties such as demand, availability of generation, and transmission lines capacity factor. Roh et al. (2009) show a stochastic long-term generation and transmission capacity planning where scenarios are created by Monte Carlo simulation for considering load uncertainties and generation and transmission availability.

Robust optimization

RO has becoming a more popular tool in the power system literature. An application of RO in planning the transition to plug-in hybrid electric vehicles was presented by Hajimiragha et al. (2011). In Street et al. (2011), a tractable novel contingency-constrained unit commitment considering $n-k$ criterion is proposed. Baringo and Conejo (2011) propose a model for constructing hourly offering curves of power producers considering price uncertainties. Jiang et al. (2012) report a study on unit commitment of thermal units under wind output uncertainty.

Chance-constrained optimization

Chance-constrained optimization has also been used in power system research. Yu et al. (2009) deal with transmission expansion planning via chance constrained optimization considering load and wind farm uncertainties. Thw work by Mazadi et al. (2009) is a an application of chance-constrained optimization in a generation expansion problem. And Zhang and Li (2011) use chance-constrained optimization in optimal power flow problems.

Regret minimization

Other recent applications include regret minimization such as Cámac et al. (2010), a transmission planning tool for addressing robustness, regret, and exposure. The work presented by Arroyo et al. (2010) is a risk-based transmission planning model that considers deliberate outages.

Adaptation cost minimization (flexibility)

Among the novel methodologies, the concept of flexibility has been introduced in Zhao et al. (2009) through the minimization of adaptation cost in transmission expansion planning. Zhao et al. (2011) also addresses the aspect of flexible transmission planning given the uncertainties of generation expansion, load, and market variables for assessing the economical benefit of distributed generation. Also, the work Maghouli et al. (2011) deals with scenario-based transmission expansion planning in a multi-objective fashion where objectives minimized are: social cost, maximum regret, and maximum adjustment cost.

CHAPTER 3. BALANCING ROBUSTNESS AND COST IN POWER SYSTEM CAPACITY EXPANSION PLANNING

3.1 Chapter overview

A RO based methodology to solve uncertain capacity expansion planning is presented. RO is a useful tool when looking for solutions that need to be robust and economically realistic under presence of multiple amounts and sources of uncertainty. Precisely, a capacity expansion planning problem selects the most cost-efficient energy production technologies to satisfy demand reliably under changing and uncertain conditions in demand, fuel prices, and resource availability, among others. To evaluate robustness, we perform different MC simulations using in- and out-of-sample uncertainties. Results of RO applied to the planning of a 13-technology portfolio power system show that RO-based plans outperform those obtained via deterministic optimization in terms of robustness at a low cost (price of robustness).

3.2 Introduction

Decision–making problems, mainly approached by optimization techniques, are traditionally solved assuming perfect knowledge of situations characterized by data. However, many of these situations are full of uncertatinty, and different instances of data can drive the optimal solution in different directions.

Research in power systems has recently focused on the ways to control and plan the grid in uncertain environments with changing conditions. More specifically, power system capacity expansion planning, understood as the selection of new power capacity from a pool of available technologies, requires a rigorous treatment of uncertainty since all the decisions have to be made based on assumptions about future states of nature.

Types of uncertainty in power system planning include fuel prices, power demand, penetration of new technologies, implementation of environmentally-related policies, and availability of renewable resources. Planning robustness is the ability of a given system plan to perform under different realizations of conditions where performance is characterized by metrics such as cost, energy price, and reliability.

A literature review for the entire dissertation was provided in 2. To that, we add the following comments on previous work that is of particular interest to the subject of this chapter.

Several efforts have incorporated uncertainty in power system planning planning. For instance, the work in Meza et al. (2007) models fuel price uncertainties. Strategies that combine the analysis of the solutions of individual scenarios in order to obtain the solution that best performs in all scenarios are explained in Linares (2002) and Firmo and Legey (2002). The work in Cámac et al. (2010), is a transmission planning tool for addressing robustness, regret, and exposure. The work presented by Arroyo et al. (2010) is a risk-based transmission planning model that considers deliberate outages. The concept of flexibility has been introduced in Zhao et al. (2009) and Zhao et al. (2011) through the minimization of adaptation cost in transmission expansion planning. And the work Maghouli et al. (2011) deals with multi-objective scenario-based transmission expansion planning.

SP has been a popular approach for handling uncertainty. For instance, the work in Lopez et al. (2007) present a two-stage stochastic programming model for dealing with random uncertainties such as demand, availability of generation, and transmission lines capacity factor. Reference Street et al. (2009) presents the selection of optimal renewable portfolios considering spot price uncertainties. The work Roh et al. (2009) shows a stochastic generation and transmission planning considering load uncertainties and generation and transmission availability.

Apart from stochastic programming, another technique to model uncertainty is RO. RO, rather than explicitly enumerating each possible outcome of uncertainty or scenario, looks for solutions that are feasible and implementable under many realizations of uncertainties Ben-Tal et al. (2009); Bertsimas and Sim (2004); Ben-Tal and Nemirovski (1998). Indeed, in the RO literature, *robustness* is achieved when the solution, once implemented, is always feasible under *any* realization of data characterized by the uncertainty set. Among the first works in RO are

Ben-Tal and Nemirovski (1998) andEl Ghaoui and Lebret (1997). References Bertsimas et al. (2011a); Ben-Tal and Nemirovski (2002) describe some of the applications of RO in finance, statistics, and enginnering.

RO has become a more popular tool in the power system literature. An application of RO in planning the transition to plug-in hybrid electric vehicles was presented by Hajimiragha et al. (2011). In Street et al. (2011), a tractable novel contingency-constrained unit commitment considering $n-k$ criterion is proposed. In Baringo and Conejo (2011) a model for constructing hourly offering curves of power producers considering price uncertainties is proposed. The work Jiang et al. (2012) reports a study on unit commitment of thermal units under wind output uncertainty.

In this work, RO is used to solve a power system capacity expansion planning model under multiple uncertainties. RO is a useful tool, given its computational tractability, when looking for solutions that need to be robust and economically realistic under presence of multiple amounts and sources of uncertainty. We consider uncertainties in demand, fuel (natural gas, coal, uranium) prices, resource availability (like wind speed, solar radiation), investment costs, and Operation and Maintenance (O&M) costs. To evaluate robustness level, we perform different MC simulations using in- and out-of-sample uncertainties. Results of RO applied to the planning of a 14-technology portfolio power system show that RO-based plans outperform those obtained via deterministic optimization in terms of risk at a low extra cost.

3.3 Robust Optimization

In this section, we show how an optimization problem with uncertain data characterized by uncertainty sets is transformed to its robust counterpart. In order to explain RO and its underlying guidelines, we use the main concepts adapted from Ben-Tal et al. (2009).

Consider the following uncertain linear program:

$$\begin{aligned} \underset{x \in \Re^n}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & a_i^\top x \le b_i, \; i = 1, \dots, m. \\ & (A, b, c) \in \mathcal{U} = \mathcal{U}^A \times \mathcal{U}^b \times \mathcal{U}^c \end{aligned} \tag{3.1}$$

$A \in \Re^{m \times n}$, $b \in \Re^m$, and $c \in \Re^n$ are arrays of uncertain parameters that lie in a convex uncertainty set $\mathcal{U}$ defined on $\Re^{m \times n} \times \Re^m \times \Re^n$ as the cartesian product of the each uncertain parameter uncertainty set. It is assumed that each $a \in [\bar{a} - \hat{a}, \bar{a} + \hat{a}]$, $b \in [\bar{b} - \hat{b}, \bar{b} + \hat{b}]$, and $c \in [\bar{c} - \hat{c}, \bar{c} + \hat{c}]$, where $\hat{a}$, $\hat{b}$ and $\hat{c}$ are the maximum variations of a, b, and c with respect to their nominal values $\bar{a}$, $\bar{b}$, and $\bar{c}$, respectively.

The RO approach deals with finding a solution to the linear program (3.1) such that it is feasible under any realization of the uncertain parameters. When the parameters are not only considered uncertain but also random, i.e. they have a probability distribution, the RO formulation still works. In fact, references Bertsimas and Sim (2004), and Ben-Tal et al. (2009) use a probability indicator to measure the level of satisfaction of the constraint.

From now on, without loss of generality, we will consider the linear program (3.1) with only one constraint of the form $a^\top x - b \le 0$. In general, a RO problem is solved by solving the following model:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \sup_{c \in \mathcal{U}^c} c^\top x \\ \text{subject to} \quad & \sup_{(a,b) \in \mathcal{U}^a \times \mathcal{U}^b} \left(a^\top x - b \right) \le 0 \end{aligned} \tag{3.2}$$

This formulation ensures that under *any* observation of uncertainty within the bounds defined by $\mathcal{U}$, the solution will be feasible. Furthermore, the minimization of the maximum value of the objective function, known as the worst-case scenario, is implemented; therefore, the optimal objective function is indeed an upper bound of the actual objective value given the uncertainty represented in c.

We are only considering uncertainty in a and b. By using affine perturbations $\eta \in \mathcal{Z}$ where $a_k = \bar{a}_k + \eta_k \hat{a}_k, \; k = 1, \dots, n$, and $b = \bar{b} + \eta_{n+1} \hat{b}$, the linear program (3.2) can be written as:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^{n} \bar{a}_k x_k - \bar{b} + \sup_{\eta \in \mathcal{Z}} \left(\sum_{k=1}^{n} \eta_k \hat{a}_k x_k - \eta_{n+1} \widehat{b} \right) \leq 0 \end{aligned} \tag{3.3}$$

A tractable representation of (3.3) is obtained when the primitive uncertainty set has special properties such as convexity. Next, we show the mathematical framework when uncertain data lies in ellipsoidal sets. Thus, the primitive uncertainty set $\mathcal{Z}$ can be parameterized as:

$$\begin{aligned} \mathcal{Z} &= \left\{ \eta \in \Re^{n+1}, \|\eta\|_2 \leq \Omega \right\} \\ &= \left\{ \eta \in \Re^{n+1}, \sqrt{\eta_1^2 + \eta_2^2 + \ldots + \eta_{n+1}^2} \leq \Omega \right\} \end{aligned}$$

The size of the uncertainty set can be controlled through Ω, which is called the uncertainty budget. Essentially, this extra parameter helps the decision maker to adjust the degree of robustness of its solution by increasing or decreasing Ω. The solution to the inner linear maximization problem in (3.3) is a point such that one of the components of the vector η is at its maximum value Ω. This optimization problem is over a convex set; therefore its solution is a global optimum. Furthermore, it has an analytical solution given by

$$\begin{aligned} \eta_k^* &= \frac{\hat{a}_k x_k}{\sqrt{\sum_{j=1}^{n} \hat{a}_j^2 x_j^2 + \hat{b}^2}} \Omega, \; k = 1, \ldots, n \\ \eta_{n+1}^* &= -\frac{\hat{b}}{\sqrt{\sum_{j=1}^{n} \hat{a}_j^2 x_j^2 + \hat{b}^2}} \Omega \end{aligned}$$

Therefore, the robust counterpart of (3.3) is

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^{n} \bar{a}_k x_k + \Omega \sqrt{\sum_{k=1}^{n} \hat{a}_k^2 x_k^2 + \hat{b}^2} \leq \bar{b} \end{aligned}$$

or equivalently

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \bar{a}^\top x + \Omega \left\| \left[\text{diag}\left(\hat{a}\right) x; \, \hat{b} \right] \right\|_2 \leq \bar{b} \end{aligned} \tag{3.4}$$

Problem (3.4), known as the RC, is a tractable representation of (3.3) since it is a second-order cone program.

Uncertainty sets can also be represented in terms of other norms as follows:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \sum_{k=1}^{n} \bar{a}_k x_k + \sup_{\|\eta\| \le \Omega} \left(\sum_{k=1}^{n} \eta_k \widehat{a}_k x_k + \eta_{n+1} \hat{b} \right) \le b \end{aligned} \tag{3.5}$$

It can be shown that the lower level optimization problem can have a compact expression in terms of the dual norm:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \bar{a}^\top x + \Omega \left\| \left[\text{diag}(\hat{a}) x;\, \hat{b} \right] \right\|_* \le \bar{b} \end{aligned} \tag{3.6}$$

Problem (3.6) is still a linear program in the case of l^1 and l^∞ norms. However, in the rest of this work, we use the l^2 norm as in (3.4). If $\Omega = 1$, the solution is robust for the uncertainty represented in the problem. If uncertainties are not known exactly, selecting the best Ω for each constraint depends on the specific application and the decision maker's objectives.

3.4 Capacity Expansion Planning

A power capacity expansion planning problem consists of determining the most cost-effective investment decisions regarding the energy portfolio in the power system while meeting future demand changes and operational constraints. The plan decides what type of technologies and where to install. In addition, we are focused on the modeling of different types and sources of uncertainties in the sector.

We recognize the nature of any planning problem is dynamic, and decisions need to be made throughout the planning horizon. However, we are using a static version since the purpose of this chapter is to illustrate the importance of addressing uncertainty issues by RO.

In this work, we are considering the capacity expansion planning of the entire US generating portfolios under uncertainty. The objective is then to obtain the least-cost and most *robust* power system portfolio considering 14 different generation technologies. The planned power system must perform satisfactorily under any possible outcome of uncertainty.

The capacity expansion planning problem is specified as the minimization of the investment, fuel, and O&M costs. Mathematically:

$$\underset{Cap, Cap^{\text{add}}, P, \theta}{\text{minimize}} \quad TC = \sum_{i\in\Phi, j\in\Psi} \tilde{I}_{i,j} Cap^{\text{add}}_{i,j} + \sum_{i\in\Phi, j\in\Psi} \left(\widetilde{OM}^{\text{f}}_{j} Cap_{i,j} + \widetilde{OM}^{\text{v}}_{j} \sum_{s\in\mathcal{S}} P_{i,j,s} h_s \right) T$$

$$+ \sum_{i\in\Phi, f\in\mathcal{F}, s\in\mathcal{S}} \widetilde{FC}_{i,f} \left(\sum_{m\in\Psi_f} H_m P_{i,m,s} h_s \right) T \tag{3.7}$$

subject to

$$\sum_{j\in\Psi} P_{i,j,s} - \sum_{k\in\Phi, l\in\mathcal{L}} b_l S_{i,l} S_{k,l} \theta_k \geq \tilde{d}_{i,s}, \ \forall i \in \Phi, s \in \mathcal{S} \tag{3.8}$$

$$b_l \left| \sum_{i\in\Phi} S_{i,l} \theta_i \right| \leq F^{\max}_l, \ \forall l \in \mathcal{L} \tag{3.9}$$

$$|\theta_i| \leq \pi, \ \forall i \in \Phi \tag{3.10}$$

$$Cap_{i,j} = Cap^{\text{existing}}_{i,j} + Cap^{\text{add}}_{i,j}, \ \forall i \in \Phi, j \in \Psi \tag{3.11}$$

$$\sum_{i\in\Phi, j\in\Psi} Cap_{i,j} \geq (1 + r) \sum_{i\in\Phi} \tilde{d}_{i,\text{peak}} \tag{3.12}$$

$$0 \leq P_{i,j,s} \leq \widetilde{CC}_{i,j,m} Cap_{i,j}, \ \forall i \in \Phi, j \in \Psi, s \in \mathcal{S} \tag{3.13}$$

$$\sum_{s\in\mathcal{S}} P_{i,j,s} h_s \leq \widetilde{CF}_{i,j} Cap_{i,j} \sum_{s\in\mathcal{S}} h_s, \ \forall i \in \Phi, j \in \Psi \tag{3.14}$$

Indexes i (and k), j, f, m, l, and s represent elements of the region set Φ, technology set Ψ, fuel set $\mathcal{F}$, fuel-based technology set Ψ_f, transmission path set $\mathcal{L}$, and LDC steps set $\mathcal{S}$ respectively. Decision variables are the capacity additions Cap^{add}, power generation P, and nodal voltage angles θ. The objective function (total cost TC) (3.7) is the sum of total investment cost and total operational cost for T years. I represents the per-MW investment cost. O&M costs are split into fixed, OM^{f}, and variable, OM^{v}. HR represents heat rate, FC the fuel cost, and h the duration (hours per year) of the LDC steps. Equation (3.8) establishes that total generation must meet demand d for every step of the LDC. Fig. 3.1 shows a three-step ($|\mathcal{S}| = 3$) LDC. It represents an arrangement of the load curve in descendent order of

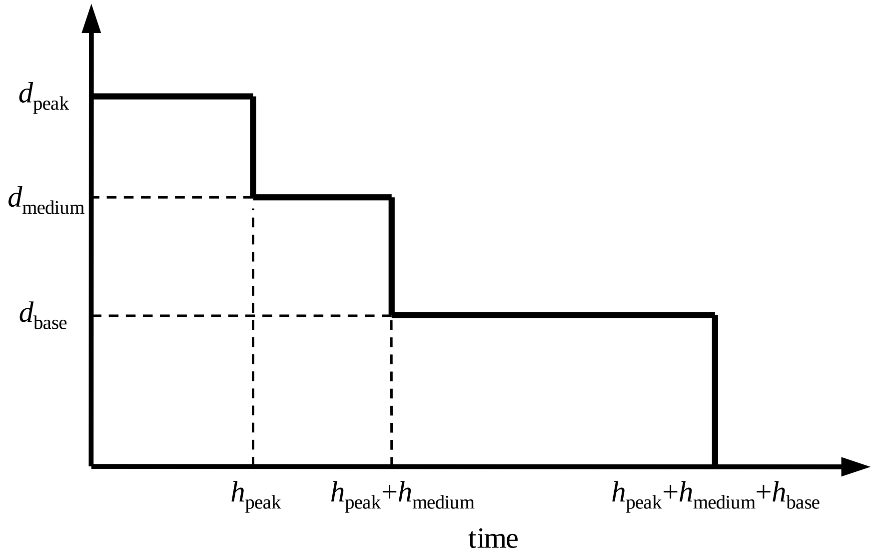


Figure 3.1: Three-step load duration curve

magnitude during a year. It is assumed that d_s is observed during h_s hours per year. The area under the LDC is the energy demand.

We use a DCOPF; so that eq. (3.8) expresses transmission flows as angular differences. The topology of the system is characterized by the incidence matrix S, and the line susceptances are represented by b. Power flows are expresed in terms of angle differences. Flow limits are imposed in (3.9). Angles (in radians) are bounded in (3.10). Equation (3.11) updates the existing installed capacity Cap^{existing}. Cap represents the portfolio of installed capacity. Constraint (3.12) imposes a capacity reserve requirement r.

The pattern of energy production varies throughout the day due to fluctuations in availability of energy resource. For example, a wind farm only delivers a portion of its rated capacity during the day since wind speeds are low; but the wind power production potential may be higher at night since wind speeds are higher. That is why we use the capacity credit CC to model the availability of energy resource as shown in (3.13).

The energy production constraint is captured in (3.14). The capacity factor CF of a plant is the average power produced in a specific period as a percentage of the maximum power the unit can produce.

3.5 Robustness testing

3.5.1 Production cost model

An optimal solution of (3.6) might not be necessarily robust for data realizations that come from outside the boundaries of the uncertainty sets. These sets are only mathematical tools useful for representing data fuzziness; but in general, it is hard to know what the actual uncertainty looks like. Therefore, it is important to evaluate how the design provided by (3.6) performs against different sizes and sources of the uncertain data.

For assessing the performance of the system, we perform MC simulations on multiple production cost models under the uncertainties that directly affect the system operation. The model minimizes production cost subject to the demand balance constraint and takes the investments in capacity Cap_j^{add} and installed capacity as given, as follows:

$$\begin{aligned}
&\underset{P,\theta,DNS}{\text{minimize}} \quad \sum_{i\in\Phi,j\in\Psi} \widetilde{OM}_j^{\text{v}} \sum_{s\in\mathcal{S}} P_{i,j,s}h_s + \sum_{i\in\Phi,f\in\mathcal{F},s\in\mathcal{S}} \widetilde{FC}_{i,f}\left(\sum_{m\in\Psi_f} H_m P_{i,m,s}h_s\right) \\
&+ \sum_{i\in\Phi,s\in\mathcal{S}} \rho_s DNS_{i,s}h_s \\
&\text{subject to} \\
&\sum_{j\in\Psi} P_{i,j,s} - \sum_{k\in\Phi,l\in\mathcal{L}} b_l S_{i,l}S_{k,l}\theta_k \geq \tilde{d}_{i,s} - DNS_{i,s},\ \forall i\in\Phi, s\in\mathcal{S} \\
&b_l\left|\sum_{i\in\Phi} S_{i,l}\theta_i\right| \leq F_l^{\max},\ \forall l\in\mathcal{L} \\
&|\theta_i| \leq \pi,\ \forall i\in\Phi \\
&0 \leq P_{i,j,s} \leq \widetilde{CC}_{i,j,s}\tilde{A}_{i,j}Cap_{i,j},\ \forall i\in\Phi, j\in\Psi,\ \forall s\in\mathcal{S} \\
&DNS_{i,s} \geq 0,\ \forall i\in\Phi, s\in\mathcal{S}
\end{aligned}$$

where $Cap_{i,j} = Cap_{i,j}^{\text{existing}} + Cap_{i,j}^{\text{add}}$. DNS represents the demand not served. It is modeled to make the production cost problem always feasible under any realization of the uncertain parameters. It is penalized in the objective function through ρ to make sure it will only be used when the planned system is not able to satisfy demand.

For a more comprehensive system robustness assessment, we assume two sources of uncertainty in which the parameters $\tilde{a}$ are independent and: 1) normally distributed with mean $\bar{a}$ and standard deviation $\hat{a}/3$, and 2) uniformly distributed in the interval $\left[\bar{a} - \hat{a}/\sqrt{3}, \bar{a} + \hat{a}\sqrt{3}\right]$, whose mean is $\bar{a}$ and standard deviation is also $\hat{a}/3$.

Additionally, we want to see how the system performs when other uncertainties, not modeled in the RO model, are actually realized. For that purpose, we simulate (aggregated) unit outages represented by availability factors $\tilde{A}_{i,j}$. Every $\tilde{A}_{i,j}$ is assumed to have a discrete distribution as follows:

$$P\left(\tilde{A}_{i,j} = A_{i,j}\right) = \begin{cases} FOR_j & \text{if } A_{i,j} = 0.8, \\ 1 - FOR_j & \text{if } A_{i,j} = 1.0 \end{cases}$$

where FOR is forced outage rate. So, with this contingency model we can say that the "aggregated" unit will operate with probability FOR; and, will operate at 80% of the credited capacity with probability $1 - FOR$. Notice that when an outage is simulated, it represents a loss of 20% in capacity of the technology in consideration, which is a high-impact contingency.

3.5.2 Robustness indicators

The following are some measures of system performance that are computed once the MC simulation is run. Expectations are estimated by sample means.

- EENS: the sample mean of the equivalent energy result of demand not attended:

$$\text{EENS} \equiv \mathbb{E}\left(\sum_{i\in\Phi, s\in\mathcal{S}} DNS_{i,s}h_s\right) \tag{3.15}$$

- Expected energy not served percentage (EENSP): the expected ratio between the energy not served and the energy demand:

$$\text{EENSP} \equiv \mathbb{E}\left(\frac{\sum_{i\in\Phi, s\in\mathcal{S}} DNS_{i,s}h_s}{\sum_{i\in\Phi, s\in\mathcal{S}} d_{i,s}h_s}\right) \times 100 \tag{3.16}$$

- Expected robustness price (ERP): the expected additional cost (in percentage) of the RO-based plan with respect to the cost of the deterministic plan:

$$\text{ERP} \equiv \mathbb{E}\left(\frac{TC}{TC^{\text{Det}}} - 1\right) \times 100 \tag{3.17}$$

where

$$TC = \sum_{i\in\Phi, j\in\Psi} \tilde{I}_{i,j} Cap_{i,j}^{\text{add}} + \sum_{i\in\Phi, j\in\Psi} \left(\widetilde{OM}_j^{\text{f}} Cap_{i,j} + \widetilde{OM}_j^{\text{v}} \sum_{s\in\mathcal{S}} P_{i,j,s} h_s \right) T$$
$$+ \sum_{i\in\Phi, f\in\mathcal{F}, s\in\mathcal{S}} \widetilde{FC}_{i,f} \left(\sum_{m\in\Psi_f} H_m P_{i,m,s} h_s \right) T$$

3.6 Results

The approach described in this work was implemented in Matlab. The RO block is performed by a Matlab-based optimization software called `CVX` Grant and Boyd (2010) useful for solving convex problems. Also, the robust counterpart is a convex and tractable representation of an uncertain optimization problem since we are using ellipsoidal uncertainty sets.

A 14-technology (CO: coal, NGCC: natural gas combined cycle, NUC: nuclear, WND: wind, WAT: hydro, SUN: solar thermal, OWND: offshore wind, ACT: advanced combustion turbine, IGCCCS: integrated gasification combined cycle with carbon sequestration, BIO: biomass, NGCCCS: natural gas combined cycle with carbon sequestration, GEO: geothermal, MSW: municipal solid waste) electricity investment portfolio is optimized considering *multiple* uncertainties in data. All data are chosen to approximately represent the features of the energy portfolio investment problem in a 5-region US system. This option was chosen because it contains geographical biases in technology attributes that are more familiar to many readers than those of a test system would be. Regions are aggregations of different states that represent the West coast (R_1), Midwest (R_2), South-Central area (R_3), Northeastern coast (R_4), and Southeastern coast (R_5).

3.6.1 Data

Figure 3.2 shows the investment costs uncertainties considered. Central values, are taken from the EIA. The lengths of each interval are based on our assumptions. In terms of investment cost, natural gas based technologies are the most attractive.

Table 3.1 shows a summary of data for region 4. The *CC* corresponding to wind and solar power are allowed to vary geographically and according to the LDC step. For example, in both

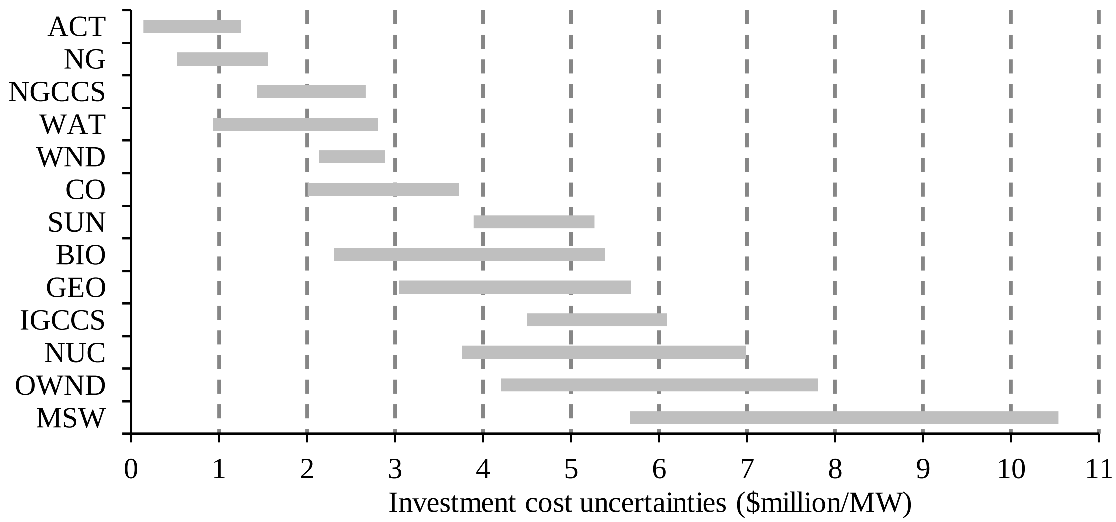


Figure 3.2: Investment costs uncertainties

Table 3.1: Summarized data for Northeastern region (R_4)

Technology	CC_{peak} (%)	CF (%)	OM^{f} ($/kW-year)	OM^{v} ($/MWh)
CO	95 ± 5	72.2 ± 8	29.6 ± 5.9	4.3 ± 0.9
NGCC	95 ± 5	40.6 ± 5	14.6 ± 2.9	3.1 ± 0.6
NUC	95 ± 5	91.1 ± 6	88.8 ± 17.8	2.0 ± 0.4
WND	20 ± 10	20.0 ± 5	28.1 ± 5.6	0.0
WAT	85 ± 10	29.4 ± 5	13.4 ± 2.7	0.0
SUN	20 ± 10	15.0 ± 5	64.0 ± 12.8	0.0
OWND	50 ± 15	35.0 ± 5	53.3 ± 10.7	0.0
ACT	95 ± 5	40.6 ± 4	6.7 ± 1.3	9.9 ± 2.0
IGCCCS	95 ± 5	72.2 ± 8	69.3 ± 13.9	8.0 ± 1.6
BIO	40 ± 20	37.3 ± 10	100.5 ± 20.1	5.0 ± 1.0
NGCCCS	95 ± 5	40.6 ± 4	30.3 ± 6.1	6.5 ± 1.3
GEO	5 ± 5	10.0 ± 10	84.3 ± 16.9	9.6 ± 1.9
MSW	40 ± 20	37.3 ± 10	373.8 ± 74.8	8.3 ± 1.7

Table 3.2: Existing capacity (GW)

Technology	R_1	R_2	R_3	R_4	R_5
CO	35.88	28.72	32.21	125.38	115.11
NGCC	88.64	17.91	97.78	106.65	143.34
NUC	9.99	5.41	6.37	44.92	39.46
WND	7.52	5.61	8.95	2.71	0.19
WAT	54.01	3.39	1.75	15.31	23.62
SUN	0.53	0.00	0.00	0.01	0.00
OWND	0.00	0.00	0.00	0.00	0.00
ACT	0.00	0.00	0.00	0.00	0.00
IGCCCS	0.00	0.00	0.00	0.00	0.00
BIO	0.02	0.00	0.02	0.04	0.02
NGCCCS	0.00	0.00	0.00	0.00	0.00
GEO	3.28	0.00	0.00	0.00	0.00
MSW	0.18	0.13	0.02	1.59	0.76

the East and West Coasts (R_1,R_4, and R_5) during peak load periods, CC of wind is allowed to vary between 10% and 30%, but it lies between 25% and 55% in the central regions (R_2 and R_3). In base load periods (nights and early mornings), it ranges between between 45% and 75% in the central regions, and between 20% and 40% in other areas. Like wind power, solar also changes by region and LDC step. Solar radiation is more intensive in the Southwestern states (part of R_1 and R_3), where its CC ranges from 30% to 50% during peak load periods. We assume it is even more intensive at medium load periods, and less intensive at base load periods. For the rest of the regions, solar power CC is as shown in Table 3.1. The model can invest in off-shore wind power everywhere except in the Midwest (R_2). The West coast (R_1) is the only candidate region for geothermal capacity investments. Table 3.1 also shows capacity factor (CF). CF of wind (WND), solar thermal (SUN), offshore wind (OWND), and geothermal (GEO) units vary among regions based on the same rationale explained for CC. From the CF standpoint, nuclear performs the best among all the 14 technologies considered. O&M costs are also shown in Table 3.1. natural gas combined cycle (NGCC) units are always among the most attractive technologies given their low O&M costs (OM^{f} and OM^{v}). Table 3.2 shows the existing installed capacity (actual data from 2008). Data was obtained from EIA.

Fig. 3.3 illustrates uncertainties in fuel cost that are used. We assume that natural gas is significantly more volatile than uranium and coal in most regions.

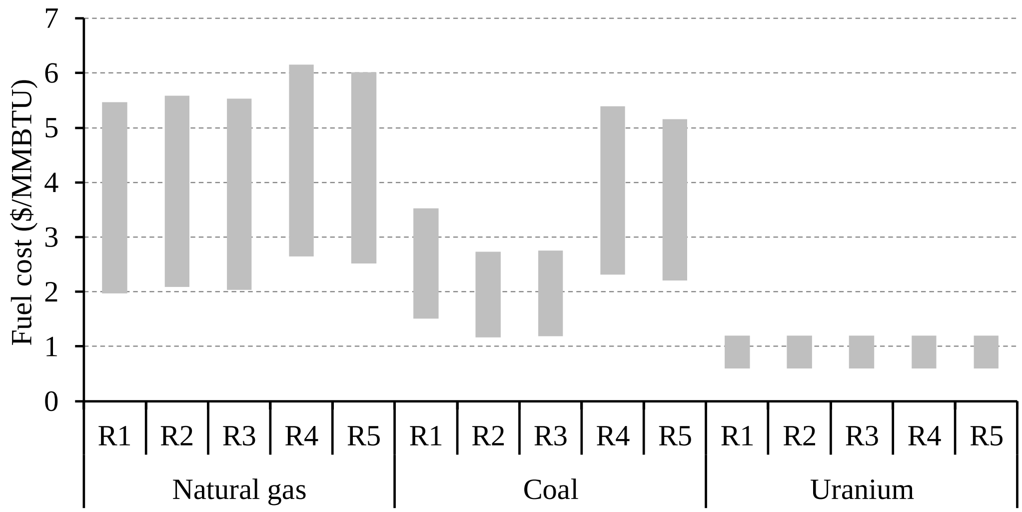


Figure 3.3: Fuel cost uncertainties for each region R_1–R_5

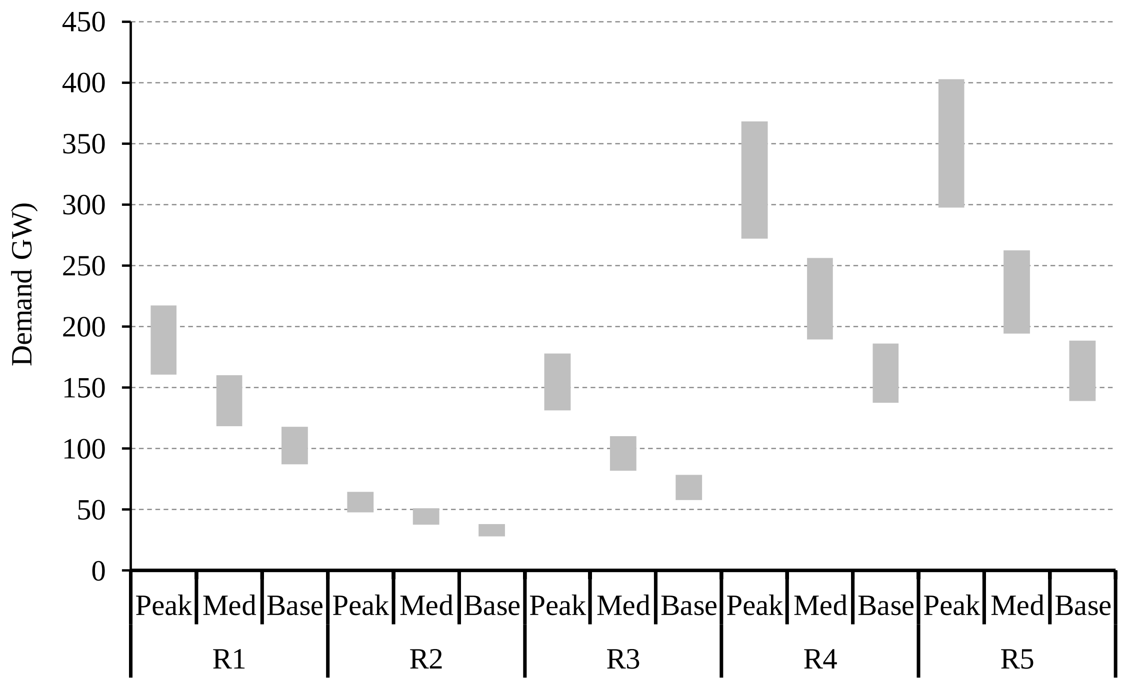


Figure 3.4: Demand steps uncertainties

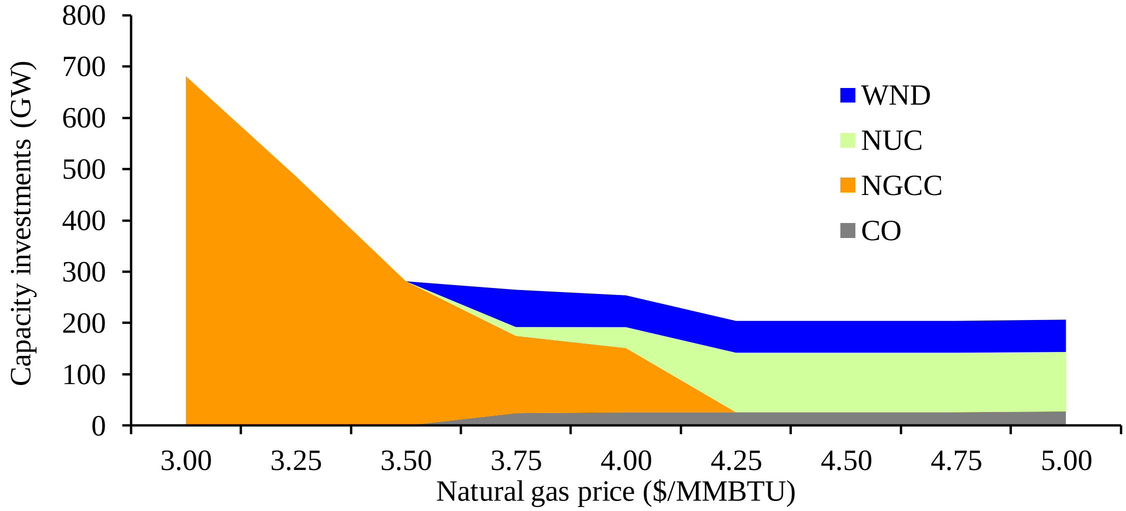


Figure 3.5: Composition of the deterministic portfolio with respect to natural gas price

Demand uncertainties are plotted in Fig. 3.4. The peak demand value is approximated by the actual 2009 value projected forty years (T) at an annual growth rate of 1%. The duration of each step of the LDC is 365, 4895, and 3500 hours per year for steps 1, 2 and 3 respectively. Medium and base demand are set such that total energy consumption corresponds to the energy demand observed in 2009.

3.6.2 Motivating the search for robustness

In this subsection, we want to motivate the use of tools able to achieve more robust solutions to uncertainties. To do so, a benchmark solution is obtained first using deterministic optimization; then we show that this solution is very sensitive to small changes in some data. The investments of the deterministic "Det" portfolio are composed mainly of NGCC in the West and East coasts, some nuclear in the East, and wind power in the Central area. However, this optimal solution is very sensitive to some uncertainties, particularly natural gas price. The regional average price used for our studies is \$4/MMBTU. Fig. 3.5 shows a sensitivity analysis of the total system capacity added with respect to gas price variation. Even fluctuations of \$1/MMBTU (from \$3.50/MMBTU to \$4.50/MMBTU) cause significant changes in the new additions of capacity. High prices favor investments in nuclear and wind power plants. Other uncertainties to which the optimal solution are sensitive are uranium price, wind investment

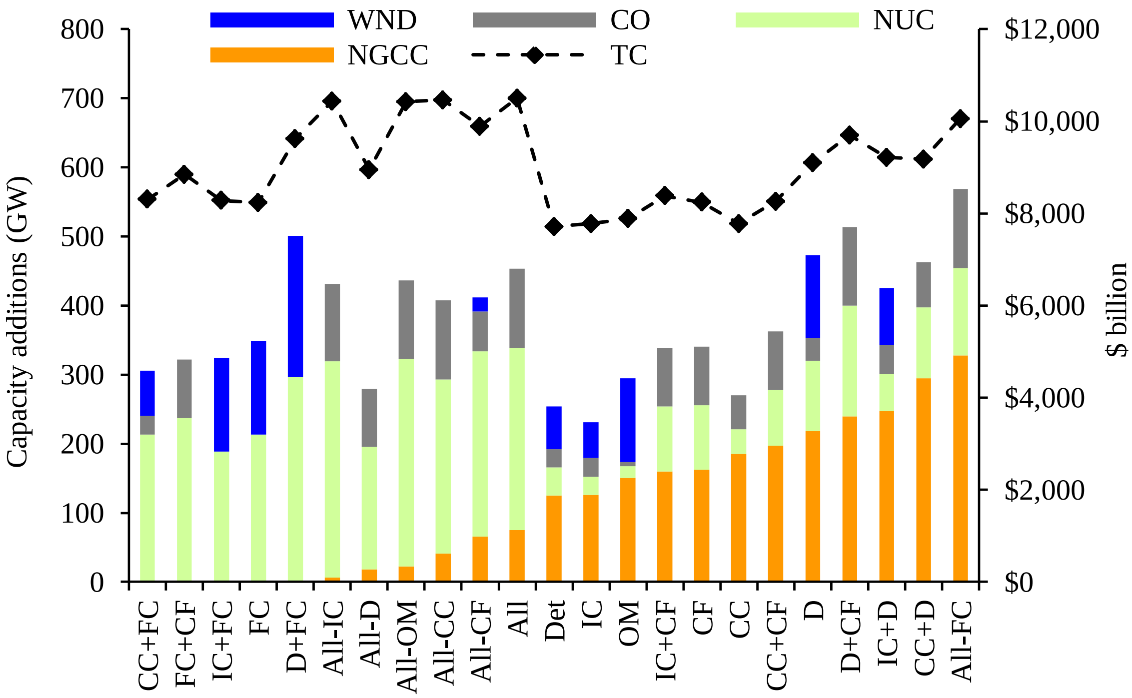


Figure 3.6: Capacity investments according to uncertainty space

cost, and NGCC capacity factor. Also, if uranium price is reduced \$0.1/MMBTU from the nominal value (\$0.9/MMBTU), nuclear investments increase dramatically whereas NGCC investments become zero. Furthermore, increases of only 2% in wind investment cost result in investments of coal rather than wind power. In addition, if NGCC capacity factor were reduced from 40% to 37%, the optimal solution replaces NGCC by investments in nuclear power.

3.6.3 Robust plans

Fig. 3.6 shows total capacity additions and total cost (TC) when uncertainties are incorporated in the robust model. Each bar represents the optimal capacity additions corresponding to different uncertainty environments. Each label describes the uncertainty space in consideration. For example, labels “Det”, “IC”, “FC”, “OM”, “D”, “CC”, “CF”, and “All” stand for deterministic, investment cost, fuel cost, O&M cost, demand, capacity credit, capacity factor, and all of the previously described uncertainties respectively. Also, a “+” or “-” sign indicates that the robust optimization is performed “with” or “without,” the uncertain parameters characterized by the acronym that follows the sign respectively.

As mentioned earlier, the "Det" case invests in NGCC, wind, nuclear, and some coal-fired capacity. These technologies are favored by their quite low investment cost (compared to other technologies). Also, wind power benefits from low O&M costs and zero fuel costs. Investments in Fig. 3.6 are sorted in increasing order according to NGCC investments. For instance, when "FC" uncertainties are considered, investments in NGCC units are minimum to avoid too much economic risk exposure. On the contrary, when "All" but the "FC" uncertainties are considered, NGCC investments are the highest. Although NGCC investment cost is higher than ACT's, NGCC is preferred because of both its lower fuel consumption (heat rate) and lower variable O&M cost.

Nuclear power is another important player in the portfolio. Almost all of the uncertainty cases are favorable for nuclear, except when "CC", "IC", and "OM" are considered. When "FC" uncertainties are modeled, nuclear prevails over NGCC because gas price is more volatile. The "Det" portfolio selects less nuclear capacity than NGCC capacity.

Wind power has low capacity credit and capacity factor; it is attractive for the model only when both demand uncertainties and those affecting the total cost are considered. Coal and wind capacity investments compete with each other. This indicates coal is attractive where wind is not, i.e., when uncertainties related to the system operation are modeled. Wind power might be more attractive if environmental constraints or policies are modeled.

Total cost of the "Det" case is $7.7 trillion whereas the "All" case is $10.5 trillion; the most robust plan would be 36.2% more costly than the "Det" plan. Each uncertainty space and its corresponding degree of robustness has a price the planner must be willing to pay. However, the tradeoffs between cost and robustness may be obtained by tuning the sizes of the uncertainty sets.

Once the robustness test is performed, the robustness indicators are computed. Fig. 3.7 shows a tradeoff between robustness price (RP), measured as the extra cost of each RO plan with respect to the cost of the deterministic plan, and EENS. In this case, the random data is assumed to be uniformly distributed. This plot suggests there might be a plan with an acceptable level of robustness (low EENS) at a reasonable RP. Our assumed expected energy demand is 5,823 TWh, and to guarantee that the ratio between Energy not served (ENS) and

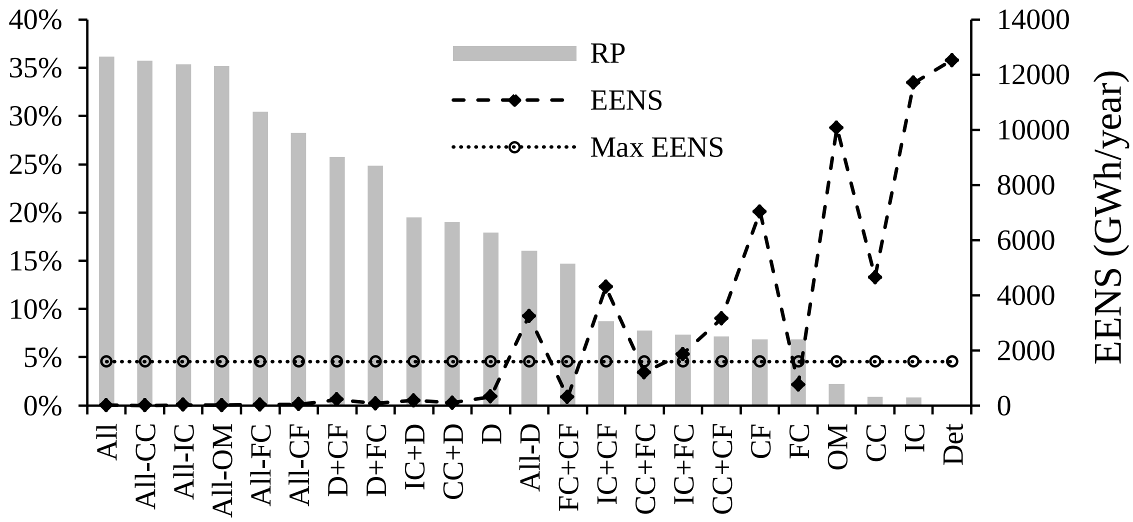


Figure 3.7: EENS and price of robustness for different uncertainty spaces

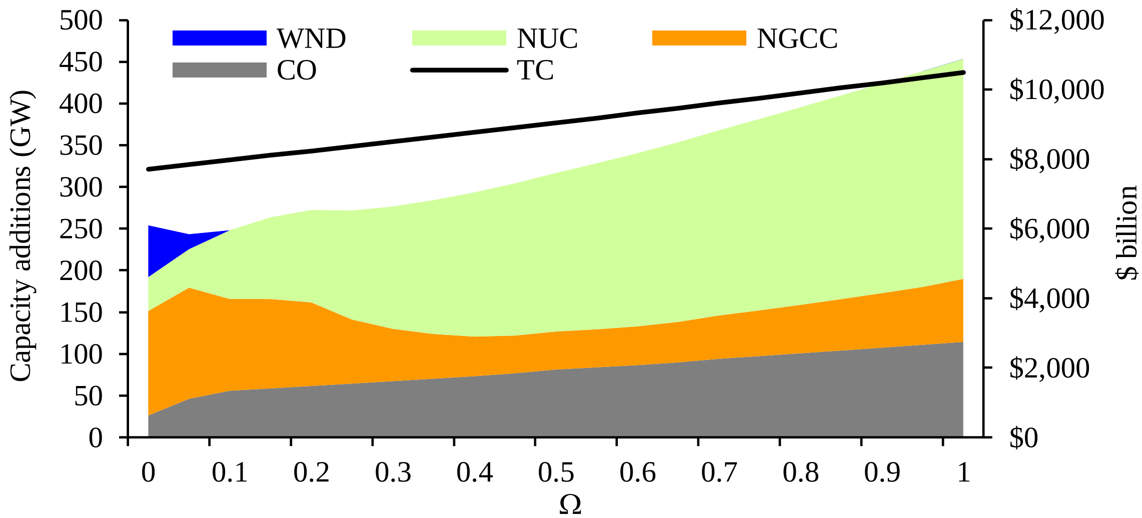


Figure 3.8: Evolution of portfolios under changes in the sizes of the uncertainty set

Table 3.3: Acceptable uncertainty budgets of RO constraints

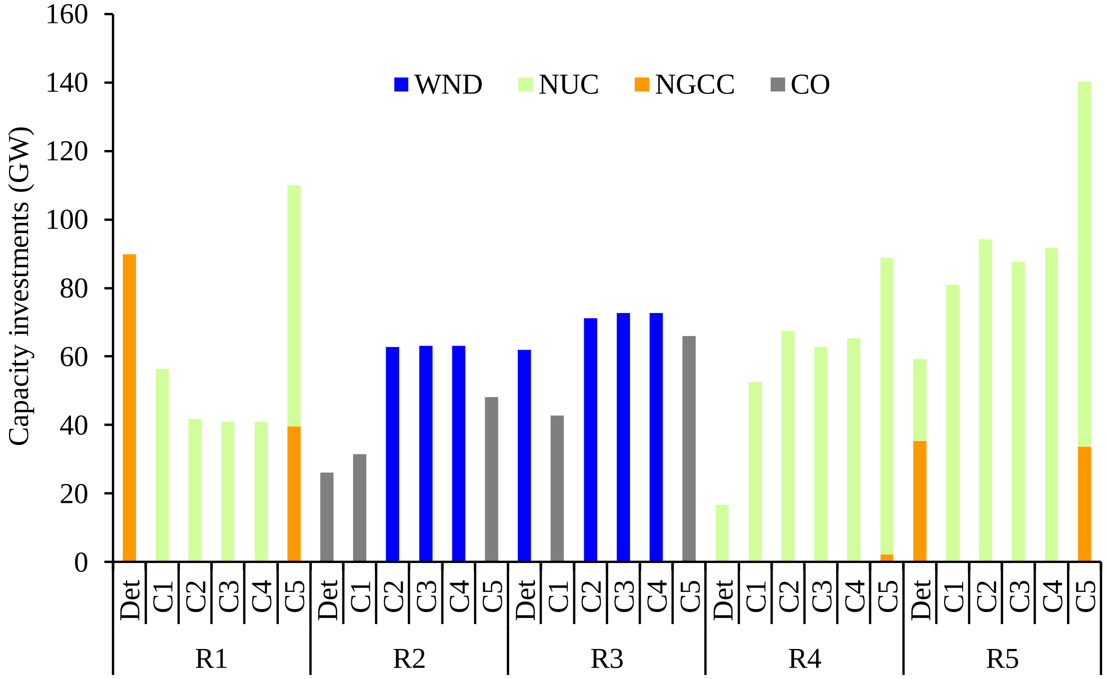
Unc. budget	C_1	C_2	C_3	C_4	C_5
Ω^{Inv}	0.0	0.0	1.1	0.0	1.0
$\Omega^{\text{O\&M}}$	0.0	0.0	0.0	0.0	1.0
Ω^{FC}	0.7	0.75	1.1	0.7	1.0
Ω^{d}	0.0	0.0	0.0	0.0	1.0
Ω^{CC}	0.0	0.75	0.0	0.0	1.0
Ω^{CF}	0.7	0.0	0.0	0.0	1.0

energy demand is equivalent to one day in ten years, then 1,594 GWh is an admissible value of EENS per year. Promising plans are those that have the potential to satisfy the EENS requirement at even lower RP.

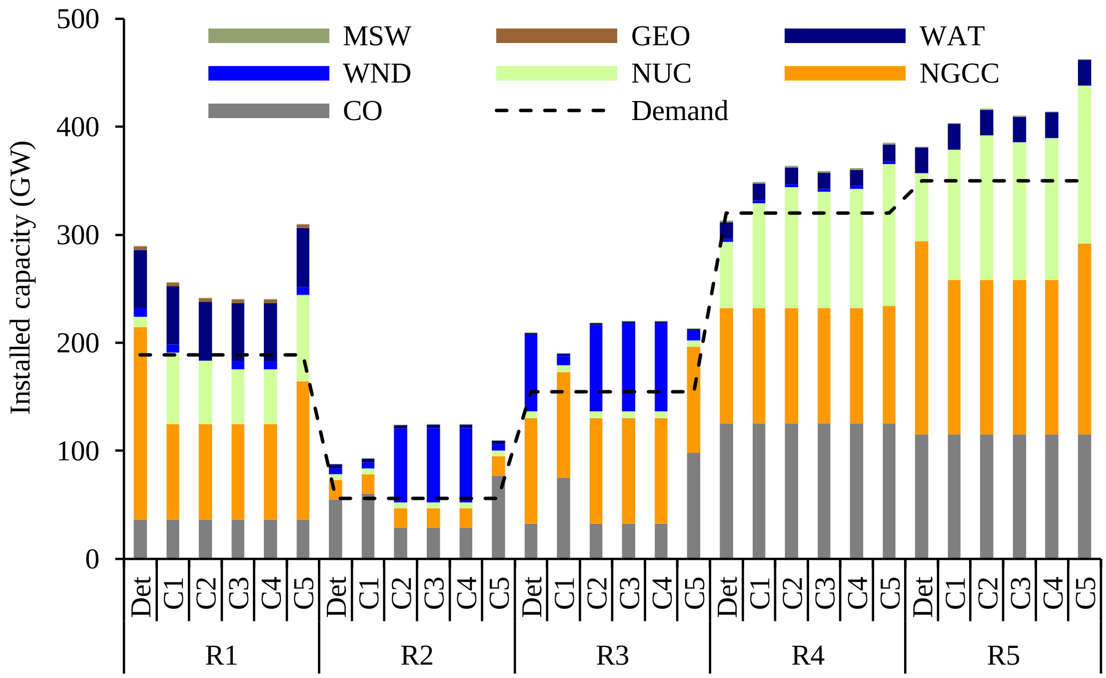
Another way to depict uncertainty influence on the solution is changing the uncertainty budget Ω. Fig. 3.8 illustrates the evolution of the robust portfolio as the uncertainty sets get bigger. Ω determines the amount of uncertainty included in uncertainty set. $\Omega = 0$ and $\Omega = 1$ correspond to the “Det” and “All” cases respectively. Unlike wind power, coal, and nuclear investments are favored by the uncertainty sets of larger size. The reason is that uncertainties in renewables production —modeled through CC and CF— affect negatively wind investments and incentivize nonrenewables penetration. Although NGCC investments are observed, gas price volatility does not allow NGCC capacity to increase significantly.

3.6.4 Candidate plans

To extend some of the results presented in part 3.6.3, we choose five portfolios from Fig. 3.6 that have the potential of lowering RP while keeping high robustness levels by refining their uncertainty set sizes. They are shown in Table 3.3 and are represented from C_1–C_5. Candidates, are obtained from the promising plans observed in Fig. 3.7 and correspond to cases “FC+CF”, “CC+FC”, “IC+FC”, “FC”, and “ALL” respectively. However, C_5 is chosen only to show what would be the RP of achieving the highest robustness levels. These cases, were those that could reduce their price of robustness and guarantee that EENSP is below to 0.03%. The values of the uncertainty budgets ($\Omega's$) were decreased (increased) manually in steps of 0.05 if the EENSP observed after performing a MC simulation was below (above)



(a) Capacity investments of candidate plans by region



(b) Installed capacity of candidate plans by region

Figure 3.9: Candidate planning solutions

0.03%. This process was repeated until a good combination of the Ωs of each constraint was found. A development of a systematic way to intensify a local search of the best Ω values would be an interesting research direction.

The investments and installed capacity of those refined plans, broken down by region, together with the "Det" plan, are plotted in Figs. 3.9a and 3.9b respectively. Investments in NGCC are observed in the "Det" and C_5 solutions, which show NGCC investment mostly in the Western Electricity Coordinating Council (WECC) (R_1) area where it is cheaper. Given the presence of fuel cost uncertainties, C_1–C_4 do not choose NGCC but nuclear given the lower volatility of uranium price. Coal power investments are observed only in the central part of the country where coal is cheaper and when significant "CF" uncertainty is considered. If it is not considered, relatively high wind power is selected. C_5 selects a mixed portfolio in regions 1, 4, and 5 (see Fig. 3.9a). The reason is that the model considers different loading conditions using the LDC, and for each condition there is an attractive technology. Thus, the model combines these technologies to choose the optimal mixed portfolio that reduces the economic risk caused by natural gas price.

Overall, nuclear and wind power are the technologies able to provide robustness to the system at a low cost. After tuning C_1–C_4, the RP of each candidate are 10.3%, 5.6%, 8%, and 5.1% respectively. The amount of power the "Det" plan selects differs from the candidates, which causes capacity reserve to be different among the plans. This lack of "smart" capacity reserve is the cause of the poor performance as we show next.

In order to evaluate how robust the designs are with respect to different amounts of uncertainty, we performed different MC simulations (with 100 iterations) using normally and uniformly distributed data with the nominal standard deviation[1] (in-sample uncertainty: N_1 and U_1), and doubling the standard deviation (out-of-sample uncertainty: N_2 and U_2). Also, recall that availability factors A, another source of out-of-sample uncertainty, are common to all of these cases. Since all data change in every iteration, we establish that a good performance level is achieved when EENSP is 0.027% (ratio between one-day energy demand and ten-year energy demand). Results are summarized in Table 3.4.

[1] the nominal standard deviation of each data parameter $\sigma_a = \hat{a}/3$

Table 3.4: MC simulation results

Indicator	Plan	Uncertainty (source and size)			
		N_1	U_1	N_2	U_2
EENS(GWh)	Det	8386	6620	13495	11958
	C_1	831	368	3797	2331
	C_2	289	291	3871	2293
	C_3	487	389	4528	2826
	C_4	365	348	4203	2564
	C_5	0	0	275	44
EENSP(%)	Det	0.142	0.112	0.224	0.200
	C_1	0.014	0.006	0.062	0.038
	C_2	0.005	0.005	0.064	0.038
	C_3	0.008	0.007	0.074	0.047
	C_4	0.006	0.006	0.069	0.043
	C_5	0.000	0.000	0.004	0.001
ERP(%)	C_1	5.96	6.07	6.62	7.35
	C_2	2.95	2.96	4.08	4.53
	C_3	2.44	2.47	3.50	3.90
	C_4	2.70	2.72	3.80	4.22
	C_5	13.47	13.60	14.86	15.84

Candidate plans show significant reductions in robustness price compared to the initial plans. Overall, C_5 always displays the best robustness indicators, and also the highest robustness price. However, C_1–C_5, balance robustness and cost efficiently for the in-sample uncertainty cases. Observe the poor performance of the "Det" plan even in the in-sample uncertainty cases. In the most optimistic case (U_1), its EENSP is equivalent to more than four days in ten years. In the N_2 case, performance of all candidates, except C_5, do not fulfill robustness requirements. Resulting events in this case are quite extreme and some might represent unlikely situations; however, if the planning objectives are very conservative, C_5 is the best solution.

Based on these simulations, building and operating a quite robust plan is only as much as 2.44% (ERP of C_3 in case N_1) of the deterministic plan, and building and operating a more robust plan under the most extreme conditions (U_2) is 15.84% (ERP of C_5). It is important to clarify that the ERP value does not consider the cost of societal consequences produced by demand curtailment (cost of ENS). If it was considered, the TC of the "Det" plan would be much higher and the resulting ERP even less (negative).

3.7 Conclusions

RO, a state-of-the-art methodology, for balancing robustness and cost in capacity expansion problem was presented. The role of RO was the design of candidate plans under different uncertainty environments to tradeoff robustness and price of robustness. And to effectively test for robustness, MC simulation of a DCOPF was used to represent the actual system operation. These tools provided signals about how to tune the amount of uncertainty to consider in the design of each robust plan. Based on the robustness tests results, it was found that the robust plans, once properly tuned, show acceptable robustness levels at low cost even for different assumptions in the uncertainty source. The traditional deterministic plan, which is remarkably sensitive to uncertainties like gas price and wind investment cost, behaved poorly under any robustness test.

CHAPTER 4. ADJUSTABLE DECISIONS FOR REDUCING THE PRICE OF ROBUSTNESS IN POWER SYSTEM CAPACITY EXPANSION PLANNING —FORMULATION

4.1 Chapter overview

This chapter proposes and implements robust optimization methodologies for making investment decisions of CEP in an environment with uncertainties in fuel prices, demand, and transmission capacity. RO and ARO techniques are used to design the robust energy portfolio. In ARO, a methodology that uses the recourse philosophy of SP in multi-stage problems plus the safe representation of uncertain constraints through RO, represents decision variables as functions of past uncertain data. Unlike SP, ARO uses uncertainty sets and avoids the explicit representation of scenario trees which makes the simulation of multiple uncertainties computationally tractable. This chapter shows both the deterministic and ARO models for the power system CEP problem; whereas Chapter 5 is dedicated to present the safe representation of the CEP under uncertainties and results. A Perfect foresight (PF), and several RO and ARO based designs are compared in the 40-year planning of a 5-region, 13-technology US energy portfolio. Unlike the PF design, RO and ARO designs display high levels of robustness at a low price.

4.2 Introduction

Uncertainties are present in any decision-making problem, and especially in those related to decisions that will be implemented several years in advance. In particular, in this chapter we are dealing with the modeling of uncertainties for the power system planning problem of making investment decisions regarding power capacity to satisfy future energy needs.

When this type of problem is solved without uncertainty considerations, results might be optimistic —cost efficient— but the quality of the investment decisions is minimal. Mathematical programming (optimization theory) was developed for problems where all the information is perfectly known; however, even simple real-life problems do not fulfill such an assumption. Intuitively, there seems to be a trade-off between optimism and quality/performance of the solution.

Planning a power system is in fact a serious problem that deserves complete attention regarding performance, which indicates a rigorous treatment of several instances of uncertainty in order to achieve, not only a cost-efficient power system, but a resilient and sustainable one. A poor performance of a power system may result in catastrophic consequences; the most severe, in terms of social and economic aspects is the curtailment of power demand of large load centers. A proper model of uncertainty is needed to help reduce the effects on the solution caused by variability of parameters, policy designs, environmental requirements, and both economic and social factors.

A long-term power system planning problem faces large amounts of uncertainty in terms of technological developments, required level of renewable energy penetration in the energy mix, carbon emission policies, cap and trade markets, fuel prices, demand behavior, electricity market evolution, renewable resource variability, and future fossil fuel reserves. The impact of each of these on the system is different with potential to dramatically change the direction of the evolution of the system.

4.2.1 Literature Review

A literature review for the entire dissertation was provided in 2. To that, we add the following comments on previous work that is of particular interest to the subject of this chapter.

All types of constraints and uncertainties are present in the long-term power system planning problem, and several approaches can be found in the literature to deal with some of them. Traditionally, sensitivity analysis, a post-optimization tool, has been an essential approach for identifying the influence on planning solutions caused by marginal changes in input data Ben-Tal and Nemirovski (1998). However, this approach does not actually protect the solution against those unforeseen uncertainties in data; but, what it does do is to provide a preliminary understanding of the effects, if any, on the planning strategy once some data change. This analysis is not enough by itself when good system performance and robustness are crucial.

For over 50 years, researchers have been thinking about solving optimization problems under presence of uncertainty; as a result, different philosophies, such as minimization of expectations, minimization of regrets, minimax of costs, and chance constrained optimization have been studied (Sahinidis (2004)). SP has been widely used as a powerful tool that does include an uncertainty model into the mathematical formulation of the problem. Basically, by making use of probability distributions of uncertain data, an stochastic program considers the minimization of the expected costs as mentioned in Birge and Louveaux (2009) and Shapiro et al. (2009); and in some applications, it also considers the minimization of risk measures (Malcolm and Zenios (1994)). For multi-stage problems, it requires the structure of a scenario tree by approximating each random variable with a fixed number of samples. Besides uncertainty approximation, the most critical drawback of realistic SP applications is the exponential growth of the scenario tree with the number of time steps, making the problem computationally intractable in general (Ben-Tal and Nemirovski (1998); Ben-Tal et al. (2009)).

Besides SP, RO has emerged as a promising research area in operations research literature like Ben-Tal and Nemirovski (1998); Ben-Tal et al. (2009); Ben-Tal and Nemirovski (1999, 2000, 2002); Bertsimas and Sim (2004, 2003); Sim (2004). Ben-Tal et al. (2009) and Ben-Tal and Nemirovski (2002) explain the RO potential applications in many disciplines. Unlike SP, RO avoids the representation of scenario trees and the sampling process to choose representative parameters; instead of that, it assumes that the uncertainty space of data is constrained to an uncertainty set and finds the best solution that is feasible for all the realizations of uncertainties that lie in the uncertainty space under consideration. Increasing robustness will definitely worsen the objective function, this is what Bertsimas and Sim (2004) define as the "price of robustness". This price is higher as long as the solution becomes more conservative (robust); however, the level of conservatism can be controlled according to the risk preferences of the decision-maker.

Several works have used RO for modeling uncertainty. Bertsimas and Sim (2003) show the mathematical formulation for combinatorial optimization and network flow problems. Alem and Morabito (2012) show an application of RO in production planning considering demand and cost uncertainties. Verderame and Floudas (2011) present an operational planning of a multisite production and distribution network considering demand and transportation time uncertainty by RO and minimization of conditional value-at-risk.

4.2.2 Adjustable RO

Since a RO solution has a significant level of conservatism, in multistage optimization it could be even more since decisions are made at the initial stage to guarantee present and future feasibility. It would be useful to allow waiting some time for analyzing some revealed information and then make the decision. This avoids extra-conservatism and improves flexibility in the investment decisions. To develop this idea, like two-stage SP, ARO splits the actual decisions (or design variables) into here-and-now and wait-and-see decision variables. The former are those decisions that will be implemented during the first stage without having observed any uncertainty realization; whereas the latter are those implemented at later stages once past uncertainties have been revealed Ben-Tal et al. (2004, 2009); Caramanis (2006). Decisions are implemented as time passes, and so, there is no real need to make all the decisions so far in advance. So, why not wait and observe the world and then make the decision?

The way decisions are made in ARO is by constructing decision rules that are functions of past or revealed information. This process allows decisions to adjust according to previous realization of uncertainties, which is similar to the idea of recourse in SP Chen et al. (2008) plus the nonanticipativity conditions. Decision rules are set only for the wait-and-see variables such that they can take corrective actions to improve the bad situations that could have happened during previous stages.

A common approximation for setting the ARO is using affine decision rules, i.e., future decisions are parameterized as affine functions of observed data. Thus, the resulting optimization problem, which is still linear under some assumptions, results in the AARC Ben-Tal et al. (2004, 2009). These type of decision rules are also referred to as linear decision rules (Chen et al. (2008)).

Among the advantages of solutions obtained using the AARC is their adaptability, reduction of the objective function (when compared to a RO), computational tractability, and robustness. Adaptability refers to solutions that can self-adjust according to the optimal decision rule. Any realization of data during some stage(s) are inputs in the computation of future decisions. For example, if gas price is considered an uncertainty in a capacity expansion problem, natural gas power capacity investments might be reduced if gas price exceeds its expectations. A natural gas investment decision rule might tell us the same rationale by using the AARC and making natural gas price part of the uncertainty affine rule. Apart from adaptability, reduction of the objective function is also possible. As long as decision variables are allowed to have more degrees of freedom, the feasibility space increases and therefore the solution is more optimal; also, adaptability avoids making future immediate expensive decisions caused by unexpected disturbances in data. Although the approach considers recourse, the resulting optimization is still tractable for finite horizon problems. Finally, robustness is another strength of this approach. Although solutions are not known so far in advance because we have to wait until uncertainties are observed, the feasibility of the solutions is guaranteed since the modeling technique is RO.

To our knowledge, the work Ben-Tal et al. (2004) was the first to introduce the idea of adjustable solutions in optimization and no ARO-related works have been reported in the literature of power systems. However, several theoretical improvements and practical applications have been reported. Reference Caramanis (2006) presents a comprehensive development of what authors call adaptable RO and illustrates an application of the approach on air traffic control under weather uncertainty. Ben-Tal et al. (2005) use an AARC to study a multistage supply chain problem under uncertainty in demand by minimization of the worst-case cost function. Adida and Perakis (2010) present some models that incorporate uncertainty in a dynamic pricing and inventory control problem; and their AARC approach outperforms dynamic programming, static RO, and stochastic programming.

This chapter proposes and implements an ARO methodology for making investment decisions of power system capacity in an uncertain environment using affine decision rules depending on information sets. Given that RO uses uncertainty sets, optimizating under multiple uncertainties is computationally tractable. Additionally, a DDP approach is presented in Chapter 5 to alternatively solve the resulting multi-stage large-scale ARO problems.

4.3 CEP

The CEP problem consists of, in general, identifying the most cost-efficient energy portfolio that will supply the energy needs of the system in a sustainable and resilient way. "Identifying" refers to finding the right amounts on investments in time and location such that future energy needs are satisfied by considering technical, societal and environmental issues, and uncertainty.

The analytical version of the CEP problem used in this work is stated as deciding how much power capacity to invest in from a set of fossil fuel and renewable generation technologies. Finding the best portfolio not only requires minimizing costs, and satisfying demand and operational limits, but also handling the variability caused by renewable generation and the risks associated with uncertainty in costs and future demand. The model we are dealing with is a multi-stage long-term investment problem that receives technical and economic signals from the annual operating problem using a LDC and modeled as a DCOPF.

4.3.1 Deterministic planning

In this section, we show the mathematical formulation of the problem. For clarifying notation, the following sets are used: stage $\mathcal{T} = \{1, \ldots, T\}$, region Φ, technology Ψ, fuel $\mathcal{F}$, fuel-based technology $\Psi_{\mathrm{F}} \subset \Psi$, nonfuel-based technology $\Psi_{\mathrm{NF}} \subset \Psi$ (and $\Psi_{\mathrm{NF}} \cap \Psi_{\mathrm{F}} = \emptyset$), transmission line $\mathcal{L}$, and LDC step $\mathcal{S}$. Unless somethng else is specified within the formulation, elements of each set are (by default) represented with the indexes t (or τ), i (or k), j, f, m, u, l, and s respectively. By definition, we use $\Psi_{\mathrm{F}} = \bigcup_{f \in \mathcal{F}} \Psi_f$, where Ψ_f is the set of technologies using fuel f. The CEP problem can be stated as:

$$\underset{Cap, Cap^{\text{add}}, P, \theta}{\text{minimize}} \quad \sum_{t,i,j} \zeta^{t-1} \left(I_{i,j,t} Cap^{\text{add}}_{i,j,t} - \mathbf{1}_{\{t=T\}} SV_{i,j} Cap_{i,j,t} \right)$$
$$+ \sum_{t,i,j} \zeta^{t-1} \left(OM^{\text{f}}_{j,t} Cap_{i,j,t} + OM^{\text{v}}_{j,t} \sum_{s} P_{i,j,s,t} h_s \right) + \sum_{t,i,f,s} \zeta^{t-1} FC_{i,f,t} \left(\sum_{m \in \Psi_f} H_m P_{i,m,s,t} h_s \right) \tag{4.1}$$

subject to

$$Cap_{i,j,t} = Cap_{i,j,t-1} + Cap^{\text{add}}_{i,j,t} - Cap^{\text{ret}}_{i,j,t},\ \forall i,j,t \tag{4.2}$$

$$Cap_{i,j,0} = Cap^{\text{existing}}_{i,j}, \forall i,j \tag{4.3}$$

$$\sum_{i,j} Cap_{i,j,t} \geq \sum_{i\in\Phi} (1+r)\, d_{i,\text{peak},t},\ \forall t \tag{4.4}$$

$$P_{i,j,s,t} \leq CC_{i,j,s} Cap_{i,j,t},\ \forall i,j,s,t \tag{4.5}$$

$$\sum_{s\in\mathcal{S}} P_{i,j,s,t} h_s \leq CF_{i,j} Cap_{i,j,t} \sum_{s\in\mathcal{S}} h_s,\ \forall i,j,t \tag{4.6}$$

$$\sum_{j} P_{i,j,s,t} - \sum_{k} b'_{i,k}\theta_k S^{\text{base}} \geq d_{i,s,t} \forall i,s,t \tag{4.7}$$

$$S^{\text{base}} b_l \left| \sum_{i} S_{i,l}\theta_i \right| \leq F^{\max}_{l,t},\ \forall l,s,t \tag{4.8}$$

$$|\theta_i| \leq \pi,\ \forall i,s,t \tag{4.9}$$

ζ is the discount factor and T is the planning horizon. The objective function (4.1) is composed of the total investment cost caused by the additions of new generating capacity Cap^{add}, the total operating cost which is the sum of the fixed (rent, water use, facility services) and variable operating (depends on actual energy production) cost. Also, the salvage value is maximized to guarantee the installed capacity has a value in the end of the planning horizon. I the overnight or investment costs of each technology, OM^{f} is the fixed O&M cost, OM^{v} is the variable O&M cost, FC is the fuel cost for coal, natural gas, and uranium; and H is the heat rate. SV is the salvage value of each unit in the end of the planning horizon and is assumed as a percentage of the investment cost.

Cap_t, the installed capacity available throughout period t, as shown in (4.2), is continuously updated balancing the capacity investments or additions Cap^{add} and the deterministic retirements of capacity $Cap^{\text{ret}}_{i,j,t}$ starting period t, and the period $t-1$ cumulated capacity $Cap_{i,j,t-1}$. At $t = 0$, capacity is the existing infrastructure at that moment as shown in (4.3).

Total capacity of the system must satisfy reserve margin r with respect to peak demand $d_{i,\text{peak},t}$ as described in (4.4). Power produced by each individual technology, especially renewables, in different periods of a typical day is bounded by the capacity credit CC as in (4.5). With CC, we consider renewable resource (wind speed, solar radiation) availability in different periods defined by the LDC steps for a typical day. For wind and solar units, this availability is much smaller than for the rest of the units.

Energy production is bounded by the capacity factor in (4.6). CF is the ratio between the average power produced in a specific period and its nominal capacity. Given the variability of renewable resources, both wind and solar CF are the lowest. h represent the duration of each LDC step in hours. For convenience, let the duration fraction of step s be $h'_s = h_s / \sum_{s' \in \mathcal{S}} h_{s'}$.

Total power generation plus (minus) imports (exports) of power, expressed as angular differences, coming at (leaving from) every region must be enough to satisfy demand at every step of the LDC as described in (4.7). The term $b'_{i,k} \equiv (\sum_l b_l S_{i,l} S_{k,l})$ relates the connectivity of nodes with voltage angles.

The power flowing by each path in the network is bounded by the thermal limits on the transmission lines. If flows are approximated and expressed in terms of angular differences, maximum (and minimum) flow constraints are as shown in (4.8). Voltage angles (in radians) are bounded according to constraints (4.9). S represents the network connectivity matrix, b line susceptances in per unit, and S^{base} the base power of the system.

Since RO-based methodologies do not allow to work with equalities (constraint (4.2)), $Cap_{i,j,t}$ can be expressed only in terms of capacity additions:

$$Cap_{i,j,t} = Cap_{i,j}^{\text{existing}} + \sum_{\tau=1}^{t} \left(Cap_{i,j,\tau}^{\text{add}} - Cap_{i,j,\tau}^{\text{ret}} \right), \ \forall i,j,t \tag{4.10}$$

Then, by using this expression, the deterministic model can be fully expressed in terms of capacity investments:

$$\begin{aligned} &\underset{Cap^{\text{add}},P,\theta}{\text{minimize}} \ \sum_{t,i,j} c_{i,j,t}^{\text{add}} Cap_{i,j,t}^{\text{add}} + \sum_{t,i,s,u} \zeta^{t-1} OM_{u,t}^{\text{v}} P_{i,s,u,t} h_s \\ &+ \sum_{t,i,s,f} \zeta^{t-1} \sum_{m \in \Psi_f} \left(OM_{m,t}^{\text{v}} + FC_{i,f,t} H_m \right) P_{i,s,m,t} h_s \end{aligned} \tag{4.11}$$

subject to

$$\sum_{i,j}\sum_{\tau=1}^{t} Cap^{\text{add}}_{i,j,\tau} \geq \sum_{i,j}\left\{Cap^{\text{existing}}_{i,j} - \sum_{\tau=1}^{t} Cap^{\text{ret}}_{i,j,\tau}\right\} + (1+r)\sum_{i} d_{i,\text{peak},t},\ \forall t \tag{4.12}$$

$$P_{i,s,j,t} - CC_{i,j,s}\sum_{\tau=1}^{t} Cap^{\text{add}}_{i,j,\tau} \leq CC_{i,j,s}\left(Cap^{\text{existing}}_{i,j} - \sum_{\tau=1}^{t} Cap^{\text{ret}}_{i,j,\tau}\right),\ \forall i,j,s,t \tag{4.13}$$

$$\sum_{s\in M} h'_s P_{i,s,j,t} - CF_{i,j}\sum_{\tau=1}^{t} Cap^{add}_{i,j,\tau} \leq CF_{i,j}\left(Cap^{\text{existing}}_{i,j} - \sum_{\tau=1}^{t} Cap^{\text{ret}}_{i,j,\tau}\right),\ \forall i,j,t \tag{4.14}$$

$$\sum_{j} P_{i,s,j,t} - \sum_{k} b'_{i,k}\theta_k S^{\text{base}} \geq d_{i,s,t},\ \forall i,s,t \tag{4.15}$$

$$S^{\text{base}} b_l \left|\sum_{i} S_{i,l}\theta_i\right| \leq F^{\max}_{l,t},\ \forall l,s,t \tag{4.16}$$

$$|\theta_i| \leq \pi,\ \forall i,s,t \tag{4.17}$$

where

$$c^{\text{add}}_{i,j,t} = \zeta^{t-1} I_{i,j,t} - \zeta^{T-1} SV_{i,j} + \sum_{\tau\geq t} \zeta^{\tau-1} OM^{\text{f}}_{j,\tau},\ \forall i,j,t$$

4.4 Adjustable Robust Optimization

A pure robust optimization is useful for static problems where performance is crucial and feasibility must be achieved under any realization of uncertainty. However, when considering multi-stage optimization two drawbacks of this approach are its relatively high cost of achieving robustness and inflexibility.

Protecting the solution against the occurrence of any modeled uncertainty is costly. As a result, the robust solution becomes conservative and the decision maker ends up paying more than necessary for not knowing the future. In multi-stage optimization problems, this situation is even worse since the uncertainty space is larger for future stages and the robust solution must be conservative to be able to handle the unknown future. This type of decision may be not attractive once the uncertainty trajectory from the initial stage to any other future stage has been observed. Rather, the decision maker could have preferred to implement a different strategy (some type of regret).

In addition to the high cost (or price of robustness as defined in Bertsimas and Sim (2004)), inflexibility is an important disadvantage of a (static) robust multi-stage optimization model. Static RO makes all of the decisions "here and now" before "seeing" what actually happened with all the uncertain parameters, which results in rigid solutions throughout the planning horizon. The inability of adjusting solutions over time lead the system under study to sub-optimal states where its performance, that could be still acceptable, might be poor given the blindness of the decision-making process. To overcome these issues, other uncertainty-related methodologies like stochastic programming, consider the idea of making "here and now" decisions only for those that will be implemented in the first stage, and the second and further stage decisions can "wait" until part of the uncertainty trajectory (branch of the scenario tree in SP) has actually been "seen."

In order to reduce the level of conservatism and increase the flexibility of the robust solution in our capacity expansion planning problem, we are using the so called ARO approach of Ben-Tal et al. (2004, 2005, 2009), also named adaptable robust optimization in Caramanis (2006). The main idea of this approach is to allow all the decision variables to arbitrarily depend on the realization of past uncertainties in a systematic way as will be shown in the next subsection.

4.4.1 Preliminaries

Like in most of multi-stage optimization problems, in the CEP problem there are two embedded subproblems: investment and operation. The optimization model presented is a joint representation of the two problems and each of them has its own decision variables. For planning, decision makers are more interested in those variables that really tell them *when*, *where* and *what* to invest in. However, to make realistic investment decisions, the investment problem must be guided by the operational problem.

Then, two types of variables can be distinguished: the actual decisions (or design) x_t^d and analysis variables x_t^a. The former are those variables that inform the decision maker about what to do; whereas the latter are optimization decision variables that do not provide decisions, but are necessary to describe the operational problem of some stage.

In ARO, the idea is to find a direct relationship between uncertainty and decisions by allowing a specific dependence between the two. In this work, as well as in other applications observed in references Ben-Tal et al. (2005, 2009); and Caramanis (2006), we chose to work with an affine relationship. That is, every optimization decision variable is allowed to affinely depend on a vector full of uncertain data to obtain the so-called decision rules or linear decision rules Chen et al. (2008).

To develop this idea, we introduce the concept of information sets. Let $\{\mathbb{I}_t\}_{t=0}^{T}$ be a sequence of information sets. Each $\mathbb{I}_t$ of them collects all the information observed on the interval $[0, t]$. By definition, $\emptyset = \mathbb{I}_0 \subset \mathbb{I}_1 \subset \ldots \subset \mathbb{I}_T$, i.e., as time passes more information is collected.

Let $f(\cdot) : \mathbb{I} \mapsto \mathbb{R}^n$ be an affine function that maps a point from the uncertainty subspace defined by the information set $\mathbb{I}$ into decision rules. Then, we set the actual decision vectors as affine decision rules of the form

$$x_t^d = f_t(\mathbb{I}_t) = \gamma_t + \sum_{u \in \mathbb{I}_t} \gamma_{t,u} \tilde{\mu}_u, \ \forall t \in \mathcal{T} \tag{4.18}$$

where γ and $\tilde{\mu}$ are the vector of coefficients of the linear decision rule and the vector of uncertainties (whose dimension is defined by the cardinality of $\mathbb{I}_t$) respectively.

Let $g(\cdot) : \mathbb{I} \mapsto \mathbb{R}^m$ be another affine function that maps a point from the uncertainty subspace defined by the information set $\mathbb{I}$ into decision rules. Then, we set the time-t analysis vectors as affine/linear decision rules of the form

$$x_t^a = g_t(\mathbb{I}_T) = \phi_t + \sum_{v \in \mathbb{I}_T} \phi_{t,v} \tilde{\nu}_v, \ \forall t \in \mathcal{T} \tag{4.19}$$

where ϕ is the vector of coefficients of the linear decision rule of the analysis variables.

Actual decisions x_t^d are allowed to affinely depend only on the information that is available up to time t. So, in order to "discover" the actual value of the decision, we need to wait until all the elements of the information set $\mathbb{I}_t$ are available. Those decisions $x_t^d, t > 0$ are called "wait and see", and those that are to be made at the initial stage x_0^d, when no information is available at all, are the "here and now" decisions. On the other hand, analysis variables x_t^a, which are not real decisions, can freely depend upon the whole information set $\mathbb{I}_T$. These are variables that define the operation of the system and do not constitute any prediction of what the actual values are going to be. In fact, if they are a function of all the information set, will have more degrees of freedom and therefore are closer to the optimal solution.

Working with affine decision rules is an approximation of the general ARO approach, which is not computationally tractable in general. When optimizing only over the coefficients ϕ^d and ϕ^a of the uncertain parameters $\tilde{u}$ and $\tilde{\nu}$ respectively the computation effort is less. However, the solutions in some problems have been reported (in reference Ben-Tal et al. (2009)) to be close to the general ARC solution.

4.4.2 Adjustable CEP model

In our power system CEP model, we distinguish the two types of variables involved in the affinely ARO formulation: 1) the actual decision/design variables correspond to investments in power capacity ($x^d = Cap^{\text{add}}$); and 2) the analysis variables correspond to power generation and nodal voltage angles ($x^a = \{P, \theta\}$). Power generation and voltage angles fully characterize the operation of the power system each year, at least in the DCOPF setup. Analysis variables provide operational signals to the investment problem. However, to get a clearer estimate of what their future values could be, production cost models must be run once the optimal infrastructure is found.

A key requirement to maintain computational tractability in ARO is to have *fixed recourse*. This means that when affine rules are plugged into the model described previously, there cannot be any product between uncertain parameters. In RO theory, uncertain parameters are characterized using affine perturbations. In order to satisfy the fixed recourse condition, we consider modeling uncertainties in $\widetilde{FC}_{k,f,t} \in \mathcal{U}^{FC}$, $\tilde{d} \in \mathcal{U}^d$, and $\tilde{F}^{\max} \in \mathcal{U}^{F^{\max}}$. $\mathcal{U}^{FC}$, $\mathcal{U}^d$, and $\mathcal{U}^{F^{\max}}$ represent the projections of the uncertainty set into the spaces of fuel cost, demand, and transmission capacity.

Investments in capacity can be affine functions of all these three uncertainties in $\mathbb{I}_t$. However, if investments depended on $\tilde{d}$ and $\tilde{F}^{\max}$, the objective function would face more risks given the variability of these parameters. Therefore, we only allow investments to be affine functions of $\widetilde{FC}$. To avoid extra computational effort the dependence relies only on $t-1$ uncertainties as follows:

$$Cap^{\text{add}}_{i,j,t} \equiv \gamma^0_{i,j,t} + \frac{\mathbf{1}_{\{t\geq 2\}}}{|\Phi|} \sum_{k,f} \gamma^{FC}_{i,j,t,f} \widetilde{FC}_{k,f,t-1}, \ \forall i,j,t \tag{4.20}$$

where $\mathcal{U}^{FC}$ represents the uncertainty set that defines fuel cost uncertainty. The resulting values of the γ coefficients may help us understand the role (at least marginally speaking) of the uncertainties in the problem; their signs tell us whether or not the impact of a specific uncertainty favors the investment of capacity, whereas their magnitudes inform us about the relative impact of the uncertainty between all the investment decisions.

Similarly, we set the decision rules of the analysis variables. Although power generation of fuel-based technologies cannot be adjustable due to fixed recourse requirements, nonfuel-based power generation does fulfill the requirement. As in the investment rule, we do not want to add more variability to the objective function allowing power generation to depend on $\tilde{d}$ and $F^{\max}$. So, for convenience, we allow each $P_{i,s,u,t}$ to depend on each $\widetilde{FC}_{k,f,t}$ and $\widetilde{FC}_{k,f,t-1}$ as follows:

$$\begin{aligned} P_{i,s,u,t} \equiv & \beta^{0}_{i,s,u,t} + \frac{\mathbf{1}_{\{t=1\}}}{|\Phi|} \sum_{k,f} \beta^{FC}_{i,s,u,1,f,1} \widetilde{FC}_{k,f,1} \\ & + \frac{\mathbf{1}_{\{t\geq 2\}}}{|\Phi|} \sum_{k,f} \sum_{\tau=t-1}^{t} \beta^{FC}_{i,s,u,t,f,\tau} \widetilde{FC}_{k,f,\tau}, \ \forall i,s,u,t \end{aligned} \tag{4.21}$$

Voltage angles, on the contrary, are allowed to depend on $\tilde{d}$ and $F^{\max}$ as follows:

$$\theta_{i,s,t} \equiv \alpha^{0}_{i,s,t} + \sum_{k,s'} \alpha^{d}_{i,s,t,k,s',\tau} \tilde{d}_{k,s',\tau} + \frac{1}{|\mathcal{L}|} \sum_{l,\tau} \alpha^{F^{\max}}_{i,s,t,\tau} \tilde{F}^{\max}_{l,\tau}, \ \forall i,s,t \tag{4.22}$$

The reason is that these coefficients (α^{d} and $\alpha^{F^{\max}}$) have potential to reduce the protection terms as will be seen in Section 5.5.2.

4.4.3 Uncertain capacity expansion plan problem

In this section, we show the uncertain version of the capacity expansion model shown in Section 4.3.

$$\underset{Cap^{\text{add}},P,\theta}{\text{minimize}} \quad \sum_{t,i,j} c^{\text{add}}_{i,j,t}\gamma^0_{i,j,t} + \sum_{t,i,s,u} \zeta^{t-1} OM^{\text{v}}_{u,t}\beta^0_{i,s,u,t}h_s + \sum_{t,i,s,f}\zeta^{t-1}\sum_{m\in\Psi_f} OM^{\text{v}}_{m,t}P_{i,s,m,t}h_s$$
$$+ \sum_{t=2}^{T}\sum_{k,f}\widetilde{FC}_{k,f,t-1}\Bigg(\sum_{i,j}\frac{c^{\text{add}}_{i,j,t}}{|\Phi|}\gamma^{FC}_{i,j,t,f} + \sum_{i,s,u}\sum_{\tau=t-1}^{t}\zeta^{\tau-1}\frac{OM^{v}_{u,\tau}}{|\Phi|}h_s\beta^{FC}_{i,s,u,\tau,f,t-1}$$
$$+ \sum_{s,m\in\Psi_f}\zeta^{t-2}H_mP_{k,s,m,t-1}h_s\Bigg) + \sum_{k,f}\widetilde{FC}_{k,f,T}\Bigg(\sum_{i,s,u}\zeta^{T-1}\frac{OM^{\text{v}}_{u,T}}{|\Phi|}h_s\beta^{FC}_{i,s,u,T,f,T}$$
$$+ \sum_{s,m\in\Psi_f}\zeta^{T-1}H_mP_{k,s,m,T}h_s\Bigg)$$

subject to

Capacity reserve:

$$\sum_{i,j}\sum_{\tau=1}^{t}\gamma^0_{i,j,\tau} + \sum_{k,f}\sum_{\tau\ge 2}\widetilde{FC}_{k,f,\tau-1}\sum_{i,j}\frac{\gamma^{FC}_{i,j,\tau,f}}{|\Phi|}$$
$$\ge \sum_{i,j}\left\{Cap^{\text{exist}}_{i,j} - \sum_{\tau=1}^{t}Cap^{\text{ret}}_{i,j,\tau}\right\} + (1+r)\sum_{i}\tilde{d}_{i,\text{peak},t},\ \forall t \tag{4.23}$$

Credited capacity of fuel-based technologies:

$$P_{i,s,m,t} - \mathbf{1}_{\{t\ge 2\}}\sum_{\tau=2}^{t}\sum_{k,f}\widetilde{FC}_{k,f,\tau-1}\frac{CC_{i,m,s}}{|\Phi|}\gamma^{FC}_{i,m,\tau,f}$$
$$\le \widetilde{CC}_{i,m,s}\left[Cap^{\text{exist}}_{i,m} - \sum_{\tau=1}^{t}\left(Cap^{\text{ret}}_{i,m,\tau} - \gamma^0_{i,m,\tau}\right)\right],\ \forall i,s,m,t \tag{4.24}$$

Credited capacity of nonfuel-based technologies:

$$\beta^0_{i,s,u,t} - \mathbf{1}_{\{t\ge 3\}}\sum_{\tau=2}^{t-1}\sum_{k,f}\widetilde{FC}_{k,f,\tau-1}\frac{CC_{i,u,s}}{|\Phi|}\gamma^{FC}_{i,u,\tau,f}$$
$$+ \sum_{k,f}\Bigg(\mathbf{1}_{\{t\ge 2\}}\frac{\widetilde{FC}_{k,f,t-1}}{|\Phi|}\left(\beta^{FC}_{i,s,u,t,f,t-1} - CC_{i,u,s}\gamma^{FC}_{i,u,t,f}\right)$$
$$+ \frac{\widetilde{FC}_{k,f,t}}{|\Phi|}\beta^{FC}_{i,s,u,t,f,t}\Bigg) - CC_{i,u,s}\sum_{\tau=1}^{t}\gamma^0_{i,u,\tau}$$
$$\le \widetilde{CC}_{i,u,s}\left(Cap^{\text{exist}}_{i,u} - \sum_{\tau=1}^{t}Cap^{ret}_{i,u,\tau}\right),\ \forall i,s,u,t \tag{4.25}$$

Maximum capacity factor of fuel-based technologies:

$$\sum_{s} h'_s P_{i,s,m,t} - \mathbf{1}_{\{t\geq 2\}} \sum_{\tau=2}^{t} \sum_{k,f} \widetilde{FC}_{k,f,\tau-1} \frac{CF_{i,m}}{|\Phi|} \gamma^{FC}_{i,m,\tau,f}$$
$$\leq CF_{i,m} \left[Cap^{\text{exist}}_{i,m} - \sum_{\tau=1}^{t} \left(Cap^{\text{ret}}_{i,m,\tau} - \gamma^{0}_{i,m,\tau} \right) \right], \ \forall i, m, t \tag{4.26}$$

Maximum capacity factor of nonfuel-based technologies:

$$\sum_{s} h'_s \beta^{0}_{i,s,u,t} - \mathbf{1}_{\{t\geq 3\}} \sum_{\tau=2}^{t-1} \sum_{k,f} \widetilde{FC}_{k,f,\tau-1} \frac{CF_{i,u}}{|\Phi|} \gamma^{FC}_{i,u,\tau,f}$$
$$+ \sum_{k,f} \Big(\mathbf{1}_{\{t\geq 2\}} \widetilde{FC}_{k,f,t-1} \Big(\sum_{s} \frac{h'_s}{|\Phi|} \beta^{FC}_{i,s,u,t,f,t-1} - \frac{CF_{i,u}}{|\Phi|} \gamma^{FC}_{i,u,t,f} \Big) + \widetilde{FC}_{k,f,t} \sum_{s} \frac{h'_s}{|\Phi|} \beta^{FC}_{i,s,u,t,f,t} \Big)$$
$$\leq \widetilde{CF}_{i,u} \left(Cap^{\text{exist}}_{i,u} - \sum_{\tau=1}^{t} \left(Cap^{\text{ret}}_{i,u,\tau} - \gamma^{0}_{i,u,\tau} \right) \right), \ \forall i, u, t \tag{4.27}$$

Nodal power balance:

$$\sum_{n} a_{i,n} \alpha^{0}_{n,s,t} - \sum_{m} P_{i,s,m,t} - \sum_{u} \beta^{0}_{i,s,u,t} + \tilde{d}_{i,s,t} \left(\sum_{n} a_{i,n} \alpha^{d}_{n,s,t,i,s,t} + \frac{1}{S_{\text{base}}} \right)$$
$$+ \sum_{k\neq i, s'\neq s, \tau\neq t} \tilde{d}_{k,s',\tau} \sum_{n} a_{i,n} \alpha^{d}_{n,s,t,k,s',\tau} + \sum_{l,\tau} \tilde{F}^{\max}_{l,\tau} \sum_{n} \frac{a_{i,n}}{|\mathcal{L}|} \alpha^{F^{\max}}_{n,s,t,l,\tau}$$
$$- \mathbf{1}_{\{t=1\}} \sum_{k,f} \frac{\widetilde{FC}_{k,f,1}}{|\Phi|} \beta^{FC}_{i,s,u,1,f,1} - \mathbf{1}_{\{t\geq 2\}} \sum_{k,f} \sum_{\tau=t-1}^{t} \frac{\widetilde{FC}_{k,f,\tau}}{|\Phi|} \beta^{FC}_{i,s,u,t,f,\tau} \leq 0, \ \forall i, s, t \tag{4.28}$$

Maximum transmission capacity:

$$\sum_{i} S_{i,l} \alpha^{0}_{i,s,t} + \tilde{F}^{\max}_{l,t} \left(\sum_{i} \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,t} - \frac{1}{b_l S^{\text{base}}} \right) + \sum_{l'\neq l, \tau\neq t} \tilde{F}^{\max}_{l',\tau} \sum_{i} \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,\tau}$$
$$+ \sum_{k,s',\tau} \tilde{d}_{k,s',\tau} \sum_{i} S_{i,l} \alpha^{d}_{i,s,t,k,s',\tau} \leq 0, \ \forall l, s, t \tag{4.29}$$

Minimum transmission capacity:

$$- \sum_{i} S_{i,l} \alpha^{0}_{i,s,t} - \tilde{F}^{\max}_{l,t} \left(\sum_{i} \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,t} + \frac{1}{b_l S^{\text{base}}} \right) - \sum_{l'\neq l, \tau\neq t} \tilde{F}^{\max}_{l',\tau} \sum_{i} \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,\tau}$$
$$- \sum_{k,s',\tau} \tilde{d}_{k,s',\tau} \sum_{i} S_{i,l} \alpha^{d}_{i,s,t,k,s',\tau} \leq 0, \ \forall l, s, t \tag{4.30}$$

Minimum investments:

$$\gamma^{0}_{i,j,t} + \mathbf{1}_{\{t\geq 2\}} \sum_{k,f} \frac{\widetilde{FC}_{k,f,t-1}}{|\Phi|} \gamma^{FC}_{i,j,t,f} \geq 0, \ \forall i, j, t \tag{4.31}$$

Voltage angle limits:

$$\left| \alpha^0_{i,s,t} + \sum_{l,\tau} \frac{\tilde{F}^{\max}_{l,\tau}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,l,\tau} + \sum_{k,s',\tau} \tilde{d}_{k,s',\tau} \alpha^d_{i,s,t,k,s',\tau} \right| \leq \pi,$$

$$\forall i, s, t \tag{4.32}$$

Minimum power generation:

$$\beta^0_{i,s,u,t} + \mathbf{1}_{\{t=1\}} \sum_{k,f} \frac{\widetilde{FC}_{k,f,1}}{|\Phi|} \beta^{FC}_{i,s,u,1,f,1} + \mathbf{1}_{\{t\geq 2\}} \sum_{k,f} \sum_{\tau=t-1}^{t} \frac{\widetilde{FC}_{k,f,\tau}}{|\Phi|} \beta^{FC}_{i,s,u,t,f,\tau} \geq 0, \ \forall i, s, u, t \tag{4.33}$$

4.5 Robust Optimization framework

In this section, we show how an optimization problem with uncertainty in parameters a (w.l.o.g.) is transformed to its RC. Consider the following uncertain linear program:

$$\begin{aligned} \underset{x \in \Re^n}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & a_i^\top x \leq b_i, \ i = 1, \ldots, m. \\ & A \in \mathcal{U} \end{aligned} \tag{4.34}$$

$A \in \Re^{m \times n}$, $b \in \Re^m$, and $c \in \Re^n$ are arrays of uncertain parameters that lie in a convex uncertainty set $\mathcal{U}$ defined on $\Re^{m \times n}$. Let $\bar{a}$ and $\hat{a}$ be vectors defined in $\Re^n$ that denote the nominal values and variability of data respectively. Bounded uncertainty can be modeled as $a = \bar{a} + \Omega\, \eta\, \hat{a}$ with each element in the uncertain vector η bounded by the primitive uncertainty set $\mathcal{Z}$. Ω is usually 1; however, risk averse decision makers tend to expand the uncertainty set and Ω can achieve values of 2–3.

A tractable RC of (4.34) is obtained when the primitive uncertainty set $\mathcal{Z}$ has special properties such as convexity. In order to maintain linearity, a reduced number of auxiliary variables, and a low level of conservatism in the solution, we use the l^1-norm type of uncertainty, i.e., $\|\eta\|_1 \leq 1$. Thus the RC we will be dealing with is

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \bar{a}_i^\top x + \Omega_i \max_{k=1,\ldots,n} (\hat{a}_{i,k}\, |x_k|) \leq b_i, \ i = 1, \ldots, m \end{aligned} \tag{4.35}$$

Problem (4.35) is a convex, tractable, and linear representation of an optimization problem containing uncertainty in data lying in $\mathcal{U}$. The RO problem obtained in Section 4.4.3 can be transformed into its AARC by using model (4.35). Indeed, in Chapter 5, the linear program (4.35) will be written in terms of the protection terms $z_i,\ i = 1, \ldots, m$ as follows:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^\top x \\ \text{subject to} \quad & \bar{a}_i^\top x + \Omega_i \, z_i \leq b_i, \ i = 1, \ldots, m \\ & -z_i \leq \hat{a}_{i,k} \, x_k \leq z_i, \ i = 1, \ldots, m \end{aligned} \tag{4.36}$$

4.6 Conclusions

In this chapter, we have presented a deterministic and an uncertain version of the CEP problem. Given the computational tractability of novel techniques like RO, we presented how the CEP solution can become more flexible by using ARO. Compared to SP, ARO presents valuable advantages like its tractability and its ability to represent continuous uncertainties rather than discrete samples. In our setup, investment decisions, power generation of nonfuel-based technologies, and voltage angles are optimization variables that are parameterized as affine functions of elements of information sets. Finally, our ARO model was presented extracting the common uncertainties in each constraint. In Chapter 5 we obtain the AARC of the CEP model, which is the safe optimization problem whose solution is fully robust against any disturbance realized within the set $\mathcal{U}$.

CHAPTER 5. ADJUSTABLE DECISIONS FOR REDUCING THE PRICE OF ROBUSTNESS IN POWER SYSTEM CAPACITY EXPANSION PLANNING —RESULTS

5.1 Chapter overview

In Chapter 4, the affinely ARO technique was presented and applied to the multi-stage power system CEP problem that considered uncertainties in fuel cost, demand, and transmission capacity. Now, with the help of RO theory we present the AARC of the power system CEP problem. Additionally, the model is developed such that the DDP algorithm can be used alternatively. Results over on the planning of a 40-year horizon, 5-region, 13-technology simplified version of the US power system are presented. Different AARC and RC soluions are compared. The key finding is based on how the AARC chooses the decision rules in the operational variables to avoid the risk-averse formulation in the RC and unnecessary extra high costs. The robustness test show that the RC is competitive only when the system faces larger uncertainties than those modeled. But, for uncertainties within the uncertainty sets, the extra total cost of the AARC solution with respect to the PF solution is only 1.7%.

5.2 Introduction

When uncertainties are parameterized in form of convex and tractable uncertainty sets, an uncertain optimization problem is nowadays solvable by RO methods. Many, although not all, of the uncertainties involved in the planning process of power systems can be effectively represented in this form when it comes to finding robust solutions. In RO theory, a solution is robust if it is feasible against any realization of the uncertainties included in the uncertainty set. Adapting the concept of robustness exposed in RO theory to power system CEP, a design is said to be 100% robust when it can satisfy demand at every moment without violating any operational constraint.

The level of robustness in the design can be controlled by selecting more or less conservative types of uncertainty sets, and/or increasing or decreasing their sizes. However, a realistic approach must be able to achieve "acceptable" levels of robustness at low cost since the cost of a fully robust system designed with box-shaped (l^{∞}-norm) uncertainty sets might be prohibitive. Then, to avoid incurring a high cost of robustness, in Chapter 4, we "arbitrarily" set up affine decision rules dependent on (l^1-norm) uncertainties. In particular, investments in capacity, power generation and voltage angles are affine functions of fuel costs, transmission capacity, and power demand. As mentioned in Chapter 4, one of the key advantages of this approach, known as ARO, is the decrease in the price of robustness compared to a traditional static/unadjustable RO application.

In this chapter we derive a "safe" version of the optimization problem, namely the affinely adjustable robust counterpart that guarantees robustness; and we present a methodology to decouple to alternatively solve this type RO problems using DDP.

5.3 AARC of our CEP Problem

The ARO model presented in Chapter 4 is now transformed into its RC by applying the RO concepts presented in Section 4.5. The way the ARO model was presented, in which terms of common uncertainties in each constrained were grouped, is useful for deriving the AARC and obtaining the corresponding constraints of the protection terms.

An interesting feature of the AARC is its high inter-temporal connectivity. Time τ constraints and the objective function contain terms from time $t = 1$ up to $t = \tau$. This structure is not convenient if a decomposition method will be implemented. In lemma 1 and corollary 2 (see appendix A), we show some results that allow us to obtain an equivalent problem where only consecutive stages are coupled. This structure does satisfy the requirements of DDP.

Then, the AARC of our CEP model becomes:

$$\underset{Cap^{\text{add}},P,\delta}{\text{minimize}} \quad \sum_{t,i,j} \zeta^{t-1} I_{i,j,t}\Big(\gamma^{0}_{i,j,t} + \frac{\mathbf{1}_{\{t\geq 2\}}}{|\Phi|}\sum_{k,f}\gamma^{FC}_{i,j,t,f}\overline{FC}_{k,f,t-1}\Big)$$
$$+\sum_{t,i,j}\zeta^{t-1}\left(OM^{\text{f}}_{j,t} - \mathbf{1}_{\{t=T\}}\sum_{i,j} SV_{i,j}\right)\overline{Cap}_{i,j,t}$$
$$+\sum_{t,i,s,u} OM^{\text{v}}_{u,t}\Big(\beta^{0}_{i,s,u,t} + \frac{\mathbf{1}_{\{t=1\}}}{|\Phi|}\sum_{k,f}\beta^{FC}_{i,s,u,1,f,1}\overline{FC}_{k,f,1}h_s$$
$$+\frac{\mathbf{1}_{\{t\geq 2\}}}{|\Phi|}\sum_{k,f}\sum_{\tau=t-1}^{t}\beta^{FC}_{i,s,u,t,f,\tau}\overline{FC}_{k,f,\tau}h_s\Big) + \sum_{t,i,s,f}\zeta^{t-1}\sum_{m\in\Psi_f}\left(H_m\overline{FC}_{i,f,t} + OM^{\text{v}}_{m,t}\right)P_{i,m,s,t}h_s$$
$$+\,\Omega^{\text{obj}}\,\sigma_T$$

subject to

Risk constraints 1 :

$$h^{\text{risk}_1}_{k,f,t} - \sigma_{t-1} + \widehat{FC}_{k,f,t-1}\left(\sum_{i,j}\frac{c^{\text{add}}_{i,j,t}}{|\Phi|}\gamma^{FC}_{i,j,t,f} + \sum_{i,s,u}\sum_{\tau=t-1}^{t}\zeta^{\tau-1}\frac{OM^{\text{v}}_{u,\tau}}{|\Phi|}h_s\beta^{FC}_{i,s,u,\tau,f,t-1}\right.$$
$$\left.+\sum_{s,m\in\Psi_f}\zeta^{t-2}H_mP_{k,s,m,t-1}h_s\right) = 0,\ \forall k,f,t\geq 2 \tag{5.1}$$

Risk constraints 2 :

$$h^{\text{risk}_2}_{k,f,t} - \sigma_{t-1} - \widehat{FC}_{k,f,t-1}\left(\sum_{i,j}\frac{c^{\text{add}}_{i,j,t}}{|\Phi|}\gamma^{FC}_{i,j,t,f} + \sum_{i,s,u}\sum_{\tau=t-1}^{t}\zeta^{\tau-1}\frac{OM^{\text{v}}_{u,\tau}}{|\Phi|}h_s\beta^{FC}_{i,s,u,\tau,f,t-1}\right.$$
$$\left.+\sum_{s,m\in\Psi_f}\zeta^{t-2}H_mP_{k,s,m,t-1}h_s\right) = 0,\ \forall k,f,t\geq 2 \tag{5.2}$$

Risk constraints T:

$$\sigma_T \geq \widetilde{FC}_{k,f,T}\left(\sum_{i,s,u}\zeta^{T-1}\frac{OM^{\text{v}}_{u,T}}{|\Phi|}h_s\beta^{FC}_{i,s,u,T,f,T} + \sum_{s,m\in\Psi_f}\zeta^{T-1}H_mP_{k,s,m,T}h_s\right),\ \forall k,f \tag{5.3}$$

Risk state equations:

$$\sigma_t - \sigma_{t-1} - h^{\text{risk}}_t = 0,\ \forall t\geq 2 \tag{5.4}$$

Reserve:

$$-\sum_{i,j}\overline{Cap}_{i,j,t} + \Omega\, z^{\text{res}}_t \leq -\sum_{i\in\Phi}(1+r)\,\bar{d}_{i,\text{peak},t},\ \forall t \tag{5.5}$$

$$z_t^{\text{res}} \geq (1+r)\,\hat{d}_{i,\text{peak},t} \tag{5.6}$$

$$z_t^{\text{res}} \geq z_t^{2\ \text{res}}$$

$$z_t^{2\ \text{res}} \geq \widehat{FC}_{k,f,\tau-1} \left| \sum_{i,j} \frac{\gamma^{FC}_{i,j,\tau,f}}{|\Phi|} \right|, \ \forall k, f, \tau \geq 2$$

Capacity update:

$$\overline{Cap}_{i,j,t} - \mathbf{1}_{\{t\geq 2\}} \overline{Cap}_{i,j,t-1} - \gamma^0_{i,j,t} - \sum_{k,f} \frac{\overline{FC}_{k,f,t-1}}{|\Phi|} \gamma^{FC}_{i,j,t,f}$$

$$= \mathbf{1}_{\{t=1\}} Cap^{\text{existing}}_{i,j} - Cap^{\text{ret}}_{i,j,t}, \ \forall i, j, t \tag{5.7}$$

Credited capacity of fuel-based technologies:

$$P_{i,s,m,t} - CC_{i,m,s} \overline{Cap}_{i,m,t} + \Omega\, z^{\text{CC acum}}_{i,s,m,t} \leq 0, \ \forall i, s, m, t$$

$$z^{\text{CC acum}}_{i,s,m,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} CC_{i,m,s} \left|\gamma^{FC}_{i,m,t,f}\right|, \ \forall i, s, m, t \geq 2$$

Credited capacity of nonfuel-based technologies:

$$\beta^0_{i,s,u,t} + \frac{1}{|\Phi|} \sum_{k,f} \sum_{\tau=\max(t-1,1)}^{t} \beta^{FC}_{i,s,u,t,f,\tau} \overline{FC}_{k,f,\tau} - CC_{i,u,s} \overline{Cap}_{i,u,t} + \Omega\, z^{\text{CC}}_{i,s,u,t} \leq 0, \ \forall i, s, u, t \tag{5.8}$$

$$z^{\text{CC}}_{i,s,u,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} \left| \beta^{FC}_{i,s,u,t,f,t-1} - CC_{i,u,s} \gamma^{FC}_{i,u,t,f} \right|, t \geq 2$$

$$z^{\text{CC}}_{i,s,u,t} \geq \frac{\widehat{FC}_{k,f,t}}{|\Phi|} \left| \beta^{FC}_{i,s,u,t,f,t} \right|, \forall i, s, u, k, f, t \tag{5.9}$$

$$z^{\text{CC acum}}_{i,s,u,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} CC_{i,u,s} \left|\gamma^{FC}_{i,u,t,f}\right|, \ \forall i, s, u, k, f, t \geq 2$$

$$z^{\text{CC}}_{i,s,u,t} - z^{\text{CC acum}}_{i,s,u,t-1} - h^{\text{CC}}_{i,s,u,t} = 0, \ \forall i, s, u, t \geq 3 \tag{5.10}$$

$$z^{\text{CC acum}}_{i,s,j,t} - z^{\text{CC acum}}_{i,s,j,t-1} - h^{\text{CC acum}}_{i,s,j,t} = 0, \ \forall i, s, u, t \geq 3$$

Maximum capacity factor of fuel-based technologies:

$$\sum_s \alpha_s P_{i,s,m,t} - CF_{i,m} \overline{Cap}_{i,m,t} + \Omega\, z^{\text{CF acum}}_{i,m,t} \leq 0, \ \forall i, m, t$$

$$z^{\text{CF acum}}_{i,m,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} CF_{i,m} \left|\gamma^{FC}_{i,m,t,f}\right|, \ \forall i, s, m, t \geq 2$$

Maximum capacity factor of nonfuel-based technologies:

$$\sum_s h'_s \beta^0_{i,s,u,t} + \frac{1}{|\Phi|} \sum_{k,f} \sum_{\tau=\max(t-1,1)}^{t} \beta^{FC}_{i,s,u,t,f,\tau} \overline{FC}_{k,f,\tau} - CF_{i,u} \overline{Cap}_{i,u,t} + \Omega\, z^{\text{CF}}_{i,u,t} \leq 0,\ \forall i,u,t \tag{5.11}$$

$$z^{\text{CF}}_{i,u,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} \left| \sum_s h'_s \beta^{FC}_{i,s,u,t,f,t-1} - CF_{i,u} \gamma^{FC}_{i,u,t,f} \right|, t \geq 2$$

$$z^{\text{CF}}_{i,u,t} \geq \frac{\widehat{FC}_{k,f,t}}{|\Phi|} \left| \sum_s h'_s \beta^{FC}_{i,s,u,t,f,t} \right|, \forall i,u,k,f,t \tag{5.12}$$

$$z^{\text{CF acum}}_{i,u,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} CF_{i,u} \left| \gamma^{FC}_{i,u,t,f} \right|,\ \forall i,u,k,f,t \geq 2$$

$$z^{\text{CF}}_{i,u,t} - z^{\text{CF acum}}_{i,u,t-1} - h^{\text{CF}}_{i,u,t} = 0,\ \forall i,u,t \geq 3 \tag{5.13}$$

$$z^{\text{CF acum}}_{i,j,t} - z^{\text{CF acum}}_{i,j,t-1} - h^{\text{CF acum}}_{i,j,t} = 0,\ \forall i,u,t \geq 3$$

Nodal power balance:

$$\sum_n b'_{i,n} \bar{\theta}_n - \sum_m P_{i,s,m,t} - \sum_u \bar{P}_{i,s,u,t} + \Omega\, z^d_{i,s,t} \leq -\bar{d}_{i,s,t},\ \forall i,s,t \tag{5.14}$$

$$z^d_{i,s,t} \geq \hat{d}_{i,s,t} \left| \sum_n b'_{i,n} \alpha^d_{n,s,t,i,s,t} + \frac{1}{S_{\text{base}}} \right|,\ \forall i,s,t \tag{5.15}$$

$$z^d_{i,s,t} \geq \hat{d}_{k,s',\tau} \left| \sum_n b'_{i,n} \alpha^d_{n,s,t,k,s',\tau} \right|,\ \forall k \neq i, s' \neq s, \tau \neq t$$

$$z^d_{i,s,t} \geq \hat{F}^{\max}_{l,\tau} \left| \sum_n \frac{b'_{i,n}}{|\mathcal{L}|} \alpha^{F^{\max}}_{n,s,t,l,\tau} \right|,\ \forall i,s,t,l,\tau$$

$$z^d_{i,s,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} \left| \beta^{FC}_{i,s,u,t,f,t-1} \right|,\ \forall i,s,t \geq 2,k,f$$

$$z^d_{i,s,t} \geq \frac{\widehat{FC}_{k,f,t}}{|\Phi|} \left| \beta^{FC}_{i,s,u,t,f,t} \right|,\ \forall i,s,t,k,f$$

Maximum and minimum transmission capacity:

$$-\sum_i S_{i,l} \bar{\theta}_{i,s,t} + \Omega\, z^{F_{\min}}_{l,s,t} \leq \frac{\bar{F}_{\max}}{b_l S^{\text{base}}},\ \forall l,s,t$$

$$\sum_i S_{i,l} \bar{\theta}_{i,s,t} + \Omega\, z^{F_{\max}}_{l,s,t} \leq \frac{\bar{F}_{\max}}{b_l S^{\text{base}}},\ \forall l,s,t$$

$$z^{F_{\min}}_{l,s,t} \geq \hat{F}^{\max}_{l,t} \left| \sum_i \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,t} + \frac{1}{b_l S^{\text{base}}} \right|,\ \forall l,s,t$$

$$z^{F_{\max}}_{l,s,t} \geq \hat{F}^{\max}_{l,t} \left| \sum_i \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,t} - \frac{1}{b_l S^{\text{base}}} \right|,\ \forall l,s,t$$

$$z^{F_{\min}}_{l,s,t},\ z^{F_{\max}}_{l,s,t} \geq \hat{F}^{\max}_{l',\tau} \left| \sum_i \frac{S_{i,l}}{|\mathcal{L}|} \alpha^{F^{\max}}_{i,s,t,\tau} \right|,\ \forall l, s, t, l' \neq l, \tau \neq t$$

$$z^{F_{\min}}_{l,s,t},\ z^{F_{\max}}_{l,s,t} \geq \hat{d}_{k,s',\tau} \left| \sum_i S_{i,l} \alpha^{d}_{i,s,t,k,s',\tau} \right|,\ \forall l, s, t, k, s', \tau$$

Minimum investments:

$$-\gamma^0_{i,j,t} - \mathbf{1}_{\{t\geq 2\}} \sum_{k,f} \frac{\widetilde{FC}_{k,f,t-1}}{|\Phi|} \gamma^{FC}_{i,j,t,f} + \Omega\, z^{\mathrm{Inv}}_{i,j,t} \leq 0,\ \forall i, j, t$$

$$z^{\mathrm{Inv}}_{i,j,t} \geq \frac{\widehat{FC}_{k,f,t-1}}{|\Phi|} \left|\gamma^{FC}_{i,j,t,f}\right|,\ \forall i, j, t \geq 2$$

Voltage angle limits:

$$-\bar{\theta}^0_{i,s,t} + \Omega\, z^{\theta_{\min}}_{i,s,t} \leq \pi,\ \bar{\theta}^0_{i,s,t} + \Omega\, z^{\theta_{\max}}_{i,s,t} \leq \pi,\ \forall i, s, t$$

$$z^{\theta_{\min}}_{i,s,t}, z^{\theta_{\max}}_{i,s,t} \geq \frac{\tilde{F}^{\max}_{l,\tau}}{|\mathcal{L}|} \left|\alpha^{F^{\max}}_{i,s,t,l,\tau}\right|,\ \forall i, s, t, l, \tau$$

$$z^{\theta_{\min}}_{i,s,t}, z^{\theta_{\max}}_{i,s,t} \geq \tilde{d}_{k,s',\tau} \left|\alpha^{d}_{i,s,t,k,s',\tau}\right|,\ \forall i, s, t, k, s', \tau$$

Minimum power generation:

$$P_{i,s,m,t} \geq 0,\ \forall i, s, m, t,$$

$$-\bar{P}_{i,s,u,t} + \Omega\, z^{P_{\min}}_{i,s,u,t} \leq 0,\ \forall i, s, u, t$$

$$z^{P_{\min}}_{i,s,u,1} \geq \frac{\widetilde{FC}_{k,f,1}}{|\Phi|} \left|\beta^{FC}_{i,s,u,1,f,1}\right|,\ \forall i, s, u$$

$$z^{P_{\min}}_{i,s,u,t} \geq \frac{\widetilde{FC}_{k,f,\tau}}{|\Phi|} \left|\beta^{FC}_{i,s,u,t,f,\tau}\right|,\ \forall i, s, u, t \geq 2, \tau = t-1, t$$

In general, the AARC adapted to DDP, is nothing but the result of using the model (4.36), lemma 1, and corollary 2 in the uncertain CEP problem presented in Section (4.4.3). The AARC is an optimization problem where the uncertain terms of the objective function are represented with their nominal value plus a risk or protection term σ_T that computes the maximum variation of the objective function for that specific uncertainty set. In the case of the constraints, there also exists a protection term z that inhibits the solution from being close to the frontier of the feasibility set; thus, the solution can guarantee the inequality always holds under any realization of uncertainty.

5.4 Dual Dynamic Programming

The derived AARC is a multi-stage problem that can take advantage of the benefits of decomposition techniques. Important references on DDP are Pereira and Pinto (1991) and Newham (2008). The main idea of DDP is to solve a large multi-stage problem by iteratively solving small single-stage optimization problems.

Like in dynamic programming (DP), DDP approximates the value of the objective function of future stages by creating Benders-like cuts to link consecutive stages. However, DP at every stage uses "many" discrete points of the variables corresponding to the previous stages. This causes the "curse of dimensionality." Rather, DDP uses a Benders-like approximation by using information of the dual variables. In this sense, the DDP does not need to discretize any decision variable and consequently does not suffer from the combinatorial explosion like DP does.

Benders decomposition is a well known methodology that solves "intelligently" several small optimizations lead by a driving (master) problem to solve the full problem. Similarly, the DDP uses the same intelligence to obtain a piece-wise linear representation of the future objective function at every stage. In fact, a two-stage DDP is exactly a Benders model as presented by Pereira and Pinto (1991). In general, the DDP algorithm uses two main processes: forward simulation and backward recursion. In the former, the solution of one stage is used to define the solution space of the preceding stage optimization. This is done from $t = 1$ up to $t = T$. The latter process is needed to find the sensitivities of the objective function generated by the solutions in the forward simulation so as to create a Benders-like cut. These two processes continue until the convergence criterion is satisfied.

5.4.1 Mathematical formulation

The AARC we developed can be represented as a linear program of the form

$$\begin{aligned}
\underset{x,y}{\text{minimize}} \quad & \sum_t \left(c_t^\top x_t + q_t^\top y_t \right) \\
\text{subject to} \quad & E_t x_t + G_t y_t = e_t - F_{t-1} x_{t-1}, \ \forall t = 1, \ldots, T \\
& A_t x_t + B_t y_t \leq b_t, \ \forall t = 1, \ldots, T \\
& x_0 \text{ given}
\end{aligned}$$

The DDP approach that solves the time-decoupled version is presented as an algorithm as follows:

Algorithm 1 DDP algorithm

Set $i = 0$, $\overline{z} = \infty$, $\underline{z} = -\infty$
while $\overline{z} - \underline{z} \geq \epsilon$ **do**
  for $t = 1$ to $T - 1$ **do**
    Forward Simulation
    if $i \geq 2$ **then**
      Solve updated version of problem (5.16) adding the following cut
      $p_t \geq p_{t,\text{bwd}}^{i-1} - (\pi_{t+1,\text{bwd}}^{i-1})^\top F_t(x_t - x_{t,\text{fwd}}^{i-1})$
    else
      Solve problem (5.16)
    end if
    Set $x_{t,\text{fwd}}^i = x_t^*$, $z_t^* = c_t^\top x_t^* + q_t^\top y_t^*$
    if $t = 1$ **then**
      Update lower bound:
      $\underline{z} = p_0^* = z_1^* + p_1^*$
    end if
  end for
  for $t = T$ to 2 **do**
    Backward Recursion
    if $t < T$ **then**
      Solve updated version of problem (5.16) adding the following cut
      $p_t \geq p_{t,\text{bwd}}^i - (\pi_{t+1,\text{bwd}}^i)^\top F_t(x_t - x_{t,\text{fwd}}^i)$
    else if $t = T$ **then**
      Solve problem (5.16)
      Update upper bound:
      $\overline{z} = \sum_{t=1}^T z_t^*$
    end if
    Compute new cut
    Set $p_{t-1,\text{bwd}}^i = p_{t-1}^*$, and $\pi_{t,\text{bwd}}^i = \pi_t^*$
  end for
  Update iteration: $i \leftarrow i + 1$
end while

Where the stage-t optimization problem for both the forward simulation and the backward recursion is given by

$$\begin{aligned} \underset{x_t, y_t}{\text{minimize}} \quad & p_{t-1} = c_t^\top x_t + q_t^\top y_t + p_t \\ \text{subject to} \quad & E_t x_t + G_t y_t = e_t - F_{t-1} x_{t-1} : \pi_t \\ & A_t x_t + B_t y_t \leq b_t \\ & p_t \in \Re \end{aligned} \tag{5.16}$$

The "updated" version of problem (5.16) is the optimization problem that, iteration after iteration, collects *all* the p-cuts defined in the algorithm.

5.5 Numerical Results

5.5.1 Data

An application of the methodology was implemented using a simplified version of the US power system aggregated in five regions: WECC, MRO, TRE-SPP, NPCC-RFC, SERC-FRCC. The candidate portfolio considered consists of 13 technologies: Dual unit advanced pulverized coal (CO), NGCC, nuclear (NUC), WND, hydro (WAT), SUN, OWND, ddvanced combustion turbine (ACT), integrated gasification combined cycle with carbon sequestration integrated gasification combined cycle with carbon sequestration (IGCCCS), biomass (BIO), natural gas combined cycle with carbon sequestration (NGCCCS), GEO, and municipal solid waste (MSW). CO, IGCCCS, NGCC, ACT, NGCCCS, and NUC form the fuel-based technology set. The rest, renewable and alternative fuel technologies like MSW, form the nonfuel-based technology set. Data was mostly obtained from EIA reports[1].

Table 5.1 shows part of the data used for the initial year. Investment costs are averaged by regions and shown in \$million/MW. Investment costs are assumed to increase at annual rates ranging from 0% in the case of WND and SUN up to 1.8% in the case of CO. For O&M data, an annual increase rate of 0.5% is used. OM^{v} and OM^{f} are expressed in \$/MWh and \$/kW-year respectively. Capacity factor changes geographically specially for renewables; for simplicity,

[1]Updated Capital Cost Estimates for Electricity Generation Plans – November 2010

Table 5.1: Data for initial stage

Tech	$\overline{I}_j$	OM^{v}	OM^{f}	$\overline{CF}_j$	$\overline{CC}_{j,s}$		
					s_{peak}	s_{med}	s_{base}
CO	2.86	4.25	59.34	72.2	95	95	95
NGCC	1.04	3.11	29.24	40.6	95	95	95
NUC	5.38	2.04	177.50	91.1	95	95	95
WND	2.51	0.00	56.14	23.1	28	21	42
WAT	3.30	0.00	26.88	29.4	85	85	85
SUN	4.58	0.00	128.00	18.4	28	47	9
OWND	6.01	0.00	106.66	32.0	40	32	64
ACT	0.69	9.87	13.40	40.6	95	95	95
IGCCCS	5.30	8.04	138.60	72.2	95	95	95
BIO	3.85	5.00	201.00	37.3	40	40	40
NGCCCS	2.05	6.45	60.50	40.6	95	95	95
GEO	10.47	9.64	168.54	18.0	10	10	10
MSW	8.11	8.33	747.52	37.3	40	40	40

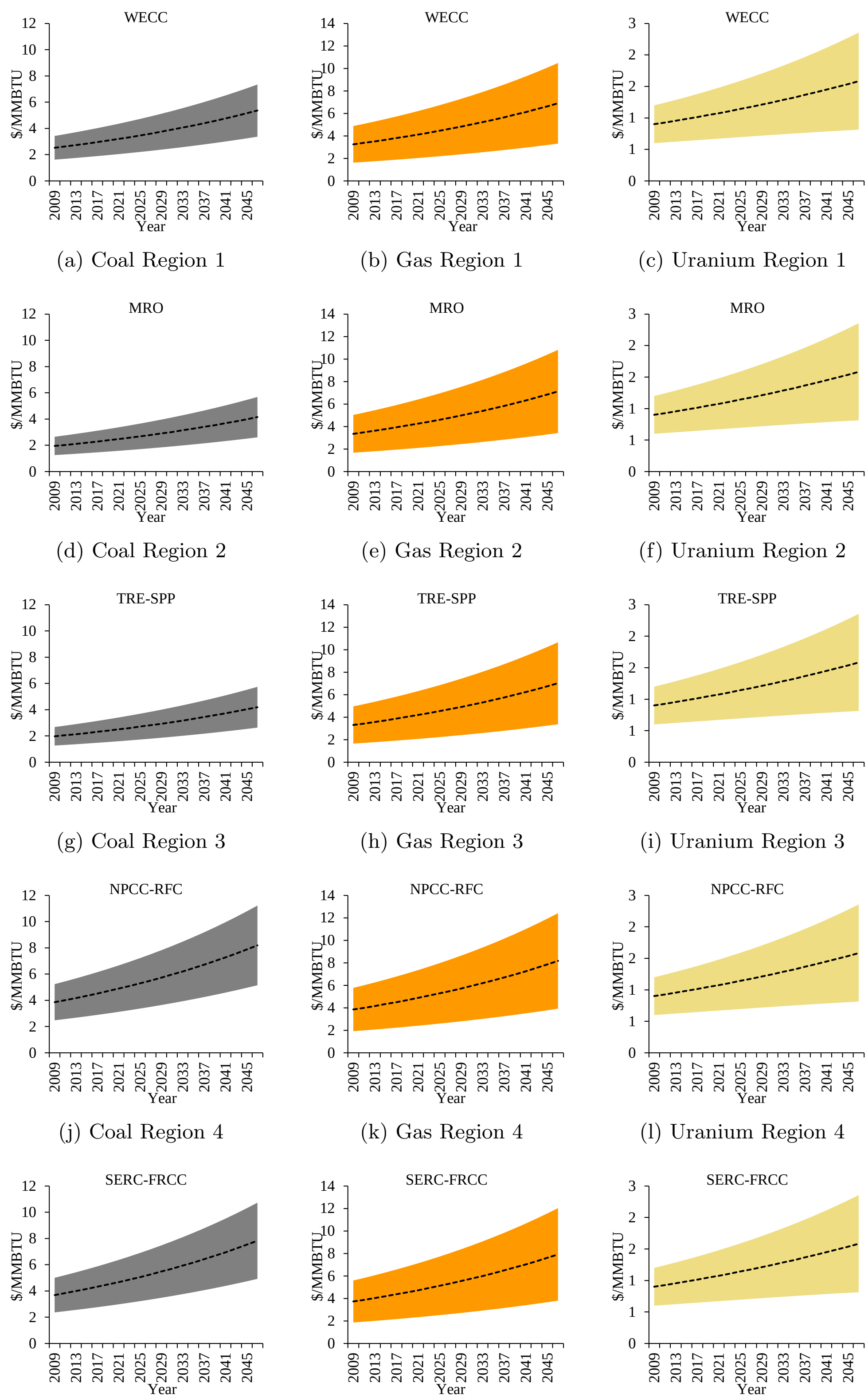
Table 5.1 shows the averages in percentage. Capacity credit is especified in percentage for each LDC step as shown in Table 5.1.

Fuel cost uncertainties of each region are plotted in Fig. 5.1. The regional average price used for gas, coal, and uranium was \$3.50/MMBTU, \$2.80/MMBTU, and \$0.90/MMBTU respectively. The rates at which these prices change per year are 2%, 2%, and 1.5% for coal, natural gas, and uranium respectively; with corresponding variabilities[2] as fractions of the nominal values of 35%, 50%, and 33% for coal, natural gas, and uranium respectively. Variabilities increase at yearly rates of 0.1% in the case of coal and gas, and 1% in the case of uranium.

Total system demand uncertainties are shown in Fig. 5.2. Nominal power peak demand of year 2009 is taken from Table 4.1 in EIA[3]. Variability with respect to the nominal value was assumed 5%. For creating a three-step LDC, we assume the durations per day of each step are 1, 13, and 10 hours for peak (late afternoon), medium (day), and base (night and early morning) load respectively. The medium and base load nominal values are such that the calculated energy demand for 2009 equals 3,950 TWh, the approximate actual 2009 energy demand in the US. At every region, the rates at which nominal values and variability grows

[2] The variability is defined as the absolute value of difference between any of the bounds and the central value of the uncertain variable

[3] Electric Power Annual 2009 – November 2010 – EIA



(a) Coal Region 1 (b) Gas Region 1 (c) Uranium Region 1

(d) Coal Region 2 (e) Gas Region 2 (f) Uranium Region 2

(g) Coal Region 3 (h) Gas Region 3 (i) Uranium Region 3

(j) Coal Region 4 (k) Gas Region 4 (l) Uranium Region 4

(m) Coal Region 5 (n) Gas Region 5 (o) Uranium Region 5

Figure 5.1: Fuel price uncertainties

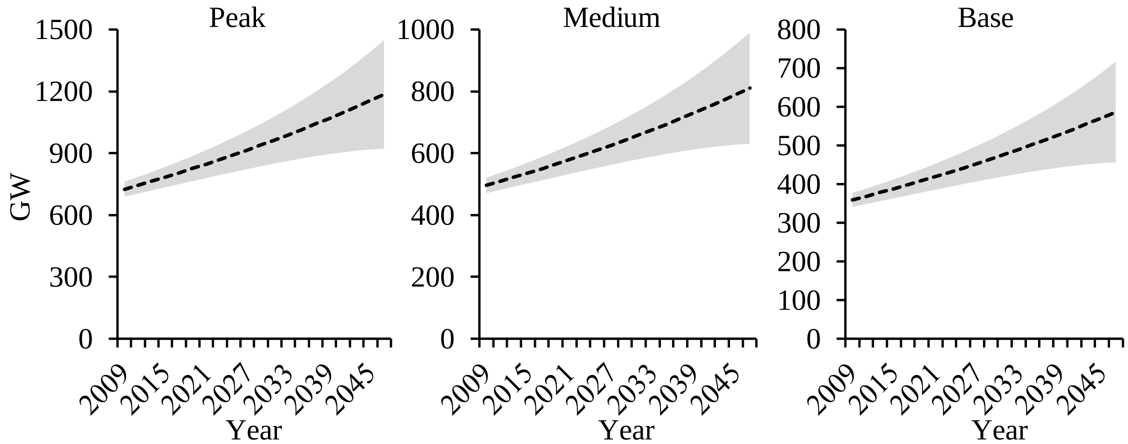


Figure 5.2: System demand uncertainties

Table 5.2: Effects of levels uncertainty on the AARC and RC

Ω^{obj}	λ	Objective function ($billion) RC	AARC
1	0.5	3,048.00	2,934.91 (-3.71%)
	1.0	3,247.38	3,041.05 (-6.35%)
	1.5	3,449.60	3,153.14 (-8.59%)
	2.0	3,654.21	3,280.05 (-10.24%)
	2.5	3,860.25	3,420.27 (-11.40%)
3	0.5	3,059.60	2,944.70 (-3.76%)
	1.0	3,271.48	3,061.91 (-6.41%)
	1.5	3,486.52	3,185.64 (-8.63%)
	2.0	3,704.75	3,224.22 (-12.97%)
	2.5	3,925.56	3,476.79 (-11.43%)
PF		2851.66	

are 1.3% and 4% respectively.

Retirements of capacity, plotted with negative values in Fig. 5.4 when units reach their lifetime are also considered. Historical investments of the US up to 2008 are used to compute the time the units have been operating and to estimate the moment of retirement. Lifetimes of technologies like CO, NGCC, NUC, WND, and WAT are assumed to be 40, 30, 60, 25, and 150 years respectively.

5.5.2 Comparing the AARC and static RC

In order to make fair conclusions regarding the AARC performance, we also obtained static RO solutions. The RC planning model is obtained when coefficients of the decision rules that multiply uncertain parameters are set to zero. For simplicity, the duration of each period is 2 years. Given this, any of the linear programs obtained were solved by the commercial optimization solver Mosek by Andersen and Andersen (2012).

Table 5.2 shows results of the RC and the AARC approach under different risk attitudes and uncertainty sizes. Ω^{obj} is used to control the level of variability of the objective function and is the key factor that determines whether the optimal investment decisions are adjustable with FC or not. And λ is a factor that multiplies all the nominal levels of variability considered in $\widehat{FC}, \hat{d}, \hat{F}^{\max}$. The PF solution, obtained in a deterministic manner assuming all the uncertain data is realized at their nominal values, is the lower bound of the RO models. Table 5.2 also shows the relative savings (in parenthesis) as a result of modeling adjustable decisions in the AARC.

In all cases, the AARC is allowed to freely adjust with fuel cost uncertainties. Recall that we allow investment affine decision rules at time t to depend on information observed at $t-1$. In the case $\Omega^{\text{obj}} = 1$, coefficients γ^{FC} and β^{FC} resulted in zero except α^d. When $\Omega^{\text{obj}} = 3$, investment decisions and power production of non-fuel based technologies are dependent on $\widetilde{FC}$, and voltage angles with $\tilde{d}$. In all the cases $\alpha^F_{\max}$ resulted different than zero; however, they did not cause any change in the overall design and objective function.

Coefficients α^d are key to provide adjustability and avoid too much conservatism in the operational problem. This dependence does not worsen robustness at all since constraint (4.15) holds for any $\tilde{d} \in \mathcal{U}^d$. These coefficients implicitly generate power flows depending on demand, which is similar to having a power flow variable for each scenario of demand in a stochastic programming setup. Fig. 5.3 shows the power flows of the RC, AARC, and PF solutions at year 40 during peak demand period. AARC power flows are represented in terms of the demand primitive uncertainties η of each region. These variable flows in turn produces a power dispatch at every bus that is not worst-case demand oriented since the AARC takes complete advantage

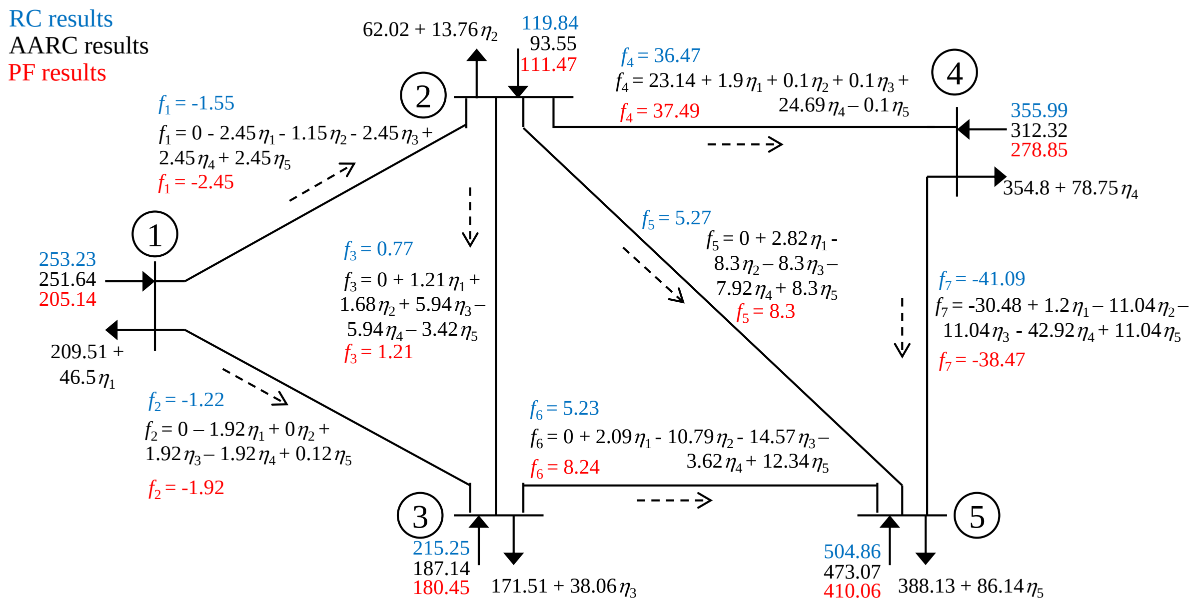


Figure 5.3: Power flows as function of primitive uncertainties of demand

of the shape of $\mathcal{U}^d$. On the other hand, power flows modeled in the RC case are unadjustable because they should be robust under any demand realization. Thus, RC solution is risk averse and assumes demand can be realized at its worst-case value at every region and at the same time. The reason is that in the RC model, each constraint (4.15) has only one uncertain term $\tilde{d}$.

AARC total generation at every region is less conservative than RC, but it is robust under any variation of the uncertainties displayed. In either the AARC or RC, generation is only informative, i.e., it does not forecast or estimate the future actual generation, but provides the appropriate operational signals under uncertainty to the investment problem.

Compared to the RC solution, what is remarkable in the AARC is its signifficant lower cost while being as robust as the RO solution (given that uncertainties actually fall in the uncertainty set). Even with the nominal uncertainty ($\lambda = 1$), the AARC objective function is 6.35% less costly than RC's. In other words, savings are above \$200 billion in 40 years (\$5,000 million/year) compared to the RC model.

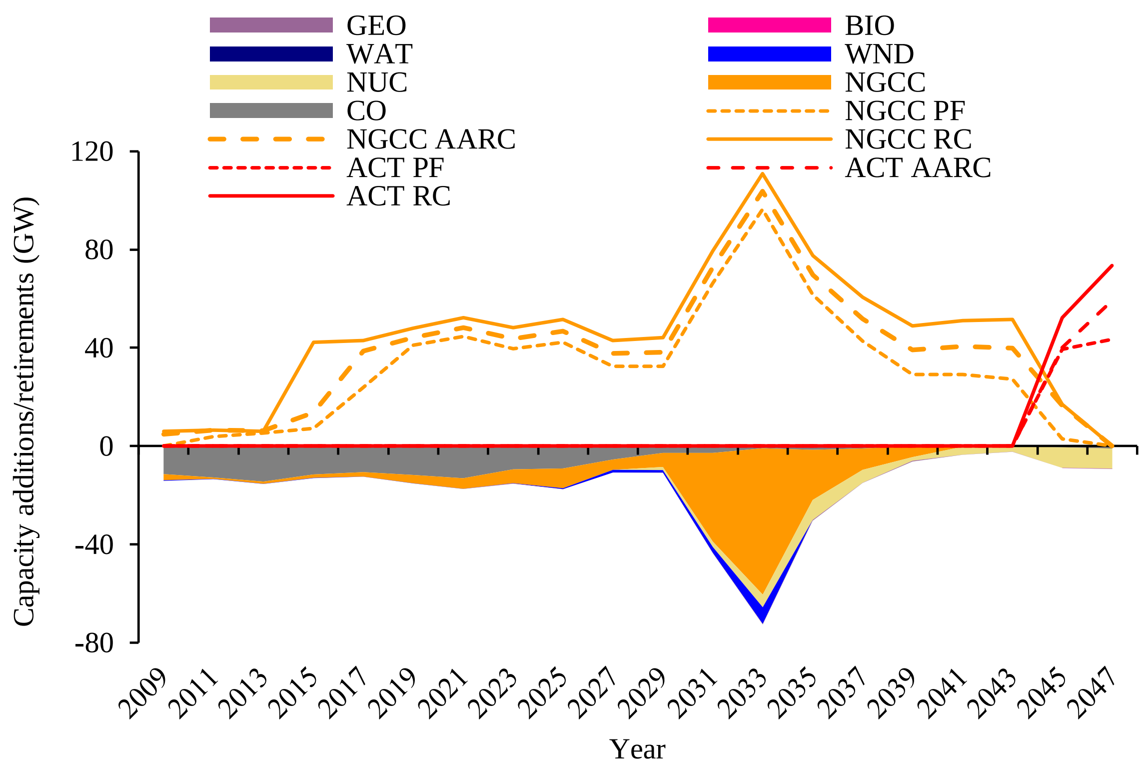


Figure 5.4: Total expected investments and retirements of the system ($\Omega^{\text{obj}} = 1,\ \lambda = 1$)

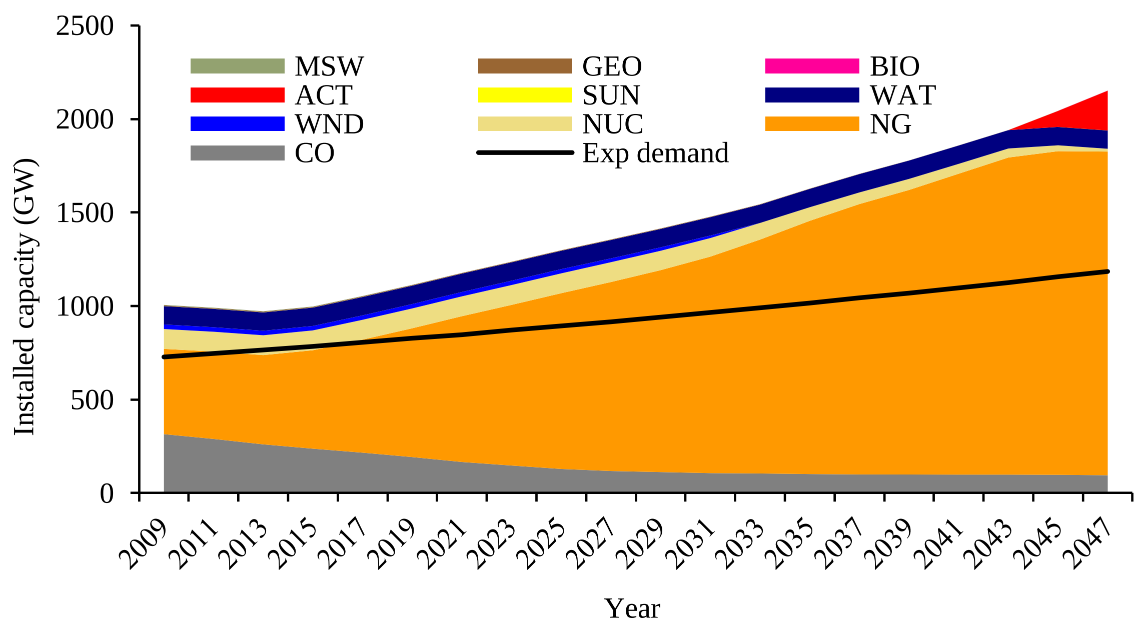


Figure 5.5: AARC installed capacity ($\Omega^{\text{obj}} = 1,\ \lambda = 1$)

5.5.3 Planning solutions

Fig. 5.4 shows the total yearly investment strategies for some of the cases of Table 5.2. All of the designs choose NGCC and ACT power plants to expand the system. The difference between all the designs relies on quantity and distribution of resources across the network. The AARC investments, although larger then PF's, are smaller than RC's. This results in lower investment costs. Retirements of NGCC capacity strongly incentivizes more investments in NGCC as depicted in Fig. 5.4. In the end of the planning horizon, ACT investments also participates in the optimal portfolios since NGCC energy production is limited by its capacity factor. Fig. 5.5 shows the evolution of the total net installed capacity of the system of the AARC. Coal and nuclear capacity decrease since no additional investments are made. The gap between total installed capacity and total demand is increasing in time given the increasing amount of uncertainty in time as well.

In terms of operation, NGCC is the key player. Its participation in the energy market is 75% in 2049. Initially, energy produced by coal units are more predominant during base load periods reaching participation close to 50%. By 2049, its participation will only reach 15% in base load periods. In peak and medium load periods, energy will be provided mostly by NGCC and ACT units.

According to our computations when Ω^{obj} is approximately greater than 2, investments become adjustable; otherwise optimal coefficientes γ are zero. As an example, below are the resulting NGCC investment (in GW) as function of $\widetilde{FC}$ (in \$/MMBTU) valid for the period 2011–2012:

$$Cap^{\text{add}}_{\text{NGCC,2011}} = \begin{pmatrix} 13.3 \\ 10.3 \\ 4.6 \\ 0.0 \\ 0.0 \end{pmatrix} + \begin{pmatrix} -0.8 & -0.6 & 3.6 \\ -7.1 & -5.1 & 32.6 \\ -3.2 & -2.3 & 14.5 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{pmatrix} \begin{pmatrix} \widetilde{FC}_{\text{coal,2009}} \\ \widetilde{FC}_{\text{gas,2009}} \\ \widetilde{FC}_{\text{ur,2009}} \end{pmatrix}$$

If coal price increases in 2009, coal power production is reduced and is replaced by NGCC production. This causes both fuel cost expenses and its associated risk to increase given that gas price is higher and more volatile than coal. Thus, for avoiding future higher risk in fuel expenses, the model inhibits NGCC investments with increments of coal price (negative sign of coefficients multiplying $\widetilde{FC}_{\text{coal},2009}$). On the other hand, increments in natural gas price reduces potential investments of NGCC units. More energy would be produced by coal than NGCC with potential gas price increments. When uranium price tends to increase, NUC energy is replaced mostly by NGCC production, encouraging more investments in this technology.

5.5.4 Price of Robustness

The term "price of robustness (PoR)", proposed in Bertsimas and Sim (2004), is used here to more carefully assess the economic performance of the RO-based planning solutions. For computing the total cost of the system, we split the investment cost from operational cost. The investment cost is computed using the investment strategies or rules in the case of AARC, and the operational cost is computed simulating the actual system operation with a DCOPF. At every year, capacity is added to and retired from the system, and the system is operated. To verify robustness of the RC and the AARC solutions, we randomly generate primitive uncertainties $\eta^{FC}, \eta^{d}, \eta^{F^{\max}}$ within the uncertainty set $\mathcal{Z}$:

$$\mathcal{Z} = \left\{ \left[\eta^{FC}; \eta^{d}; \eta^{F^{\max}}\right] : \left\| \left[\eta^{FC}; \eta^{d}; \eta^{F^{\max}}\right] \right\|_1 \leq 1 \right\}$$

Table 5.3 shows mean values (and standard deviations) of the objective function and the PoR. For each of the 100 simulations, both the RC and AARC models are feasible as expected. Based on this economic assessment, objective function values are lower than those of Table 5.2. This happens because in the design of the system, the model has to be feasible under the most hazardous combination of *all* the uncertainties in $\mathcal{U}$; whereas in the DCOPF, a power dispatch is computed for random realizations within $\mathcal{U}$. For computing the mean objective function in the PF case, a deterministic planning tool is run for each of the 100 realizations of the uncertainties. The objective functions of both the AARC and RC are lower because the operation is simulated for each realization. In average, the PoR of the AARC is less than 1.65% in the case of nominal uncertainties.

Table 5.3: Price of Robustness and (Standard Error) in ($billion)

Ω^{obj}	λ	Objective function			PoR (%)	
		PF	AARC	RC	AARC	RC
1	0.5	2851.5 (1.19)	2869.9 (1.17)	2896.9 (1.17)	0.65 (0.05)	1.59 (0.01)
	1.0	2851.3 (2.43)	2898.2 (2.37)	2951.5 (2.39)	1.65 (0.01)	3.51 (0.02)
	1.5	2851.1 (3.66)	2930.3 (3.53)	3015.7 (3.64)	2.78 (0.02)	5.77 (0.03)
	2.0	2850.8 (4.93)	2969.6 (4.72)	3086.5 (5.01)	4.17 (0.03)	8.27 (0.04)
	2.5	2850.6 (6.26)	3016.2 (6.00)	3161.0 (6.50)	5.81 (0.04)	10.89 (0.06)
3	0.5	2851.5 (1.19)	2871.1 (1.10)	2897.0 (1.17)	0.69 (0.01)	1.60 (0.01)
	1.0	2851.3 (2.43)	2900.8 (2.20)	2952.1 (2.39)	1.74 (0.02)	3.53 (0.02)
	1.5	2851.1 (3.66)	2934.0 (3.28)	3017.3 (3.63)	2.91 (0.03)	5.83 (0.03)
	2.0	2850.8 (4.93)	2975.2 (4.35)	3088.5 (5.00)	4.37 (0.06)	8.34 (0.04)
	2.5	2850.6 (6.26)	3024.5 (5.53)	3163.3 (6.49)	6.10 (0.08)	10.97 (0.06)

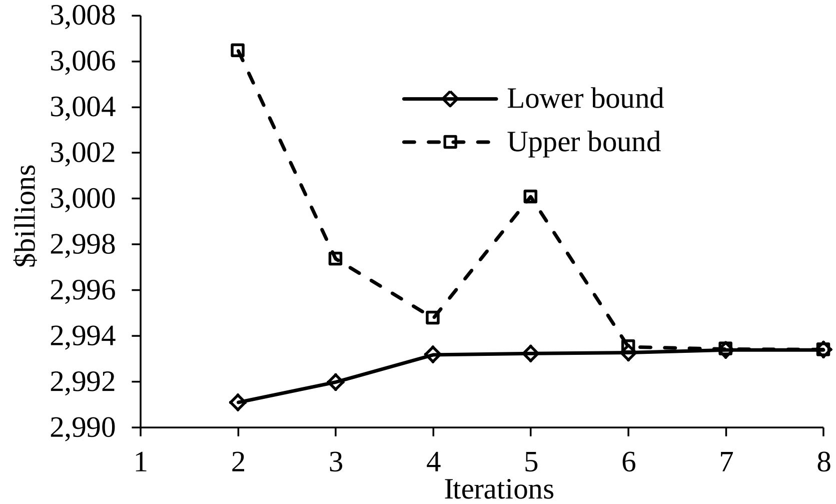


Figure 5.6: DDP convergence

5.5.5 DDP performance

The algorithm of DDP was implemented for solving the larger problems. In Fig. 5.6, we show the convergence process of the DDP algorithm when we solved the AARC model with $\Omega^{\text{risk}} = 1$. We verified that the solution achieved by the DDP technique is exactly the same as that found directly as a standard linear program. When the entire horizon of 40 years is discretized in one-year periods, the optimization problem had 124,599 constraints and 25,039 variables. With DDP, we solve 80 small linear programs per iteration that are solved quickly by using the hot-start capabilities of Mosek. Basis information of previous iterations are saved to initialize the problems of future iterations. In this case, the algorithm achieved a relative gap of 3.41×10^{-6}, and an absolute gap 0.01 in only seven iterations.

5.5.6 Robustness testing

For evaluating robustness of each design, we perform a 100-iteration MC simulation to simulate the system operation throughout the first ten years of the planning horizon. We think beyond ten years from now the designs become obsolete and investment decisions should be reviewed. A DCOPF is also run for each of the 2-year periods; it takes the system capacity as given and computes both the power generation of each technology and the voltage angles for each step of the LDC for every realization of the uncertainties. Demand, transmission

Table 5.4: Robustness test results: mean (standard error)

Ω^{obj}	λ	ENS (TWh)		ENSP (%)	
		AARC	RC	AARC	RC
1	1.0	59.0 (75)	10.8 (16)	0.141 (0.179)	0.026 (0.039)
	1.5	19.8 (39)	0.3 (1)	0.047 (0.093)	0.001 (0.003)
	2.0	4.0 (13)	0.0 (0.0)	0.010 (0.030)	0.00 (0.00)
3	1.0	43.0 (52)	10.8 (16)	0.103 (0.124)	0.03 (0.039)
	1.5	11.6 (23)	0.3 (1)	0.028 (0.055)	0.00 (0.003)
	2.0	1.7 (8)	0.0 (0.0)	0.004 (0.018)	0.00 (0.0)
PF		220.7 (146)		0.529 (0.35)	

capacity, and fuel cost data are randomly generated using uniform distributions *doubling* ($\lambda = 2$) their uncertainty space. Generating independent and uniformly distributed random numbers is equivalent to use uncertainties that come from box-shaped uncertainty sets, which contains much more uncertain elements than our l_1-norm set. Thus, our designs with $\lambda = 1$ and $\lambda = 1.5$ are exposed to multiple combinations of uncertainties that were not considered both in the RC and AARC models. For each MC iteration, the PF and RC system capacity does not change; but, it does in the case of the adjustable robust plans with $\Omega^{\text{obj}} = 3$ according the optimal coefficients γ^{FC} and the fuel cost data generated at each iteration.

Table 5.4 shows robustness indicators obtained from the simulation. Indicators are the sample mean and standard error of ENS and Energy not served percentage (ENSP) over the first 10-year period respectively. EENSP is computed as the average ratio between total ENS and total energy demand realized over the 10-year period.

A quite conservative criterion we use for determining the level of robustness is when ENSP $< 1/3650$. If the system curtailment ratio is equal or less than the energy demand of one day in ten years, it is said to be robust. As expected, the designs obtained with $\lambda = 1$ exceed this threshold except the RC solution. The RC solutions perform well even when in the case of $\lambda = 1.5$. The PF design displays the poorest robustness indicators. The AARC solution displays robustness when $\lambda = 2$. Based on results presented in Table 5.2, this design is still less costly than the RC solutions that satisfy the robustness threshold. In addition, the price to pay for its robustness is only 4.37% as shown in Table 5.3.

5.6 Conclusions

Given the computational tractability of novel techniques, we presented how the AARC solution of the CEP problem can be robust at lower PoR than the RC. In our setup, investment decisions, power generation, and voltage angles are parameterized as affine functions of fuel cost, demand, and transmission capacity. Through ARO the formulation, it is not required obtaining discrete samples of uncertainties; therefore, the explicit representation of scenario trees is avoided, making the consideration of multiple uncertainties computationally tractable. Additionally, the model was presented in a way that the DDP algorithm could be used in the case it was difficult to solve it as a standard linear program. Results over on the planning of a 40-year horizon, 5-region, 13-technology simplified version of the US power system were presented. Different AARC and RC soluions were compared.

The decision rules of the operational variables —voltage angles— were key to reduce the PoR by avoiding the installment of additional unnecessary capacity to satisfy uncertain demand; and the investment decisions are more adjustable as long as the risk in fuel costs expenses are more penalized. The robustness test shows that high PoR of the RC is justified only when the system faces larger uncertainties than those modeled; otherwise, the AARC is more competitive. But, for uncertainties within the uncertainty sets, the PoR is only 1.7%.

Multi-stage planning is significantly benefited from ARO. Not only did the PoR can be lowered; but also the decision rules depending on available information provide intuition about what the effects of individual uncertainties are on the decisions. These features make of ARO an interesting tool for long-term decision-making problems.

CHAPTER 6. MAXIMIZING FUTURE FLEXIBILITY IN ELECTRIC GENERATION PORTFOLIOS

6.1 Chapter overview

This chapter presents a methodology to obtain flexible future capacity expansion plans under diverse types and sources of uncertainty classified as global and local. Global (or high-impact) uncertainties allow us to create scenarios that train the flexibility-based planning model; whereas local uncertainties allow us to create uncertainty sets that model the imperfect knowledge of each global uncertainty. Our methodology, rather than chooosing the most flexible plan among a set of candidate solutions, actually designs a flexible system that is less sensitive to the choice of scenarios. In addition to minimizing the investment and operational cost, the model minimizes its future adaptation cost to the conditions of other identified scenarios via ARO. Results obtained with our methodology in a 5-region US system under a 40-year planning horizon show how a flexible system adapts to future high-impact uncertainties is achievable at reasonable low costs with a low number of adaptation actions. A folding horizon process where global uncertainties are guided by Markov chains was performed to measure the degree of flexibility of the system and its cost under multiple operation conditions.

6.2 Introduction

The goal of the power system planning application reported in Chapters 3, 4, and 5 that models uncertainties is to achieve high levels of robustness. A robust system is seen as one able to perform well under any —small or large— perturbations resulting from the realizations of uncertainties. However, achieving good robustness can be costly.

To avoid constructing robust but expensive power system expansion plans, we propose designing the expansion plans by maximizing future flexibility. According to the valuable works in Zhao et al. (2009) and Zhao et al. (2011), a system is flexible when it can be adapted cost-efficiently to *any* of the conditions characterizing the identified scenarios. A flexible system will not necessarily provide robust strategies to all scenarios; rather, it will be a design that is able to migrate and adjust, if needed, to the conditions of any scenario. The need is motivated by market and/or regulatory changes in the future, making it likely that an adaptation or migration process needs to take place.

Authors of paper Zhao et al. (2009) argue that the period between the plan design and its actual implementation is long, and consequently, the revealed scenario once the system starts to operate differs from the one considered for the initial design. As a result, the planning solution has to be adapted in a timely and cost-effective way to the new conditions. The cost of adapting a planning solution to a scenario is the cost of the new investment decisions that guarantee the system will meet all of the requirements under the realized scenario. The flexible plan is *selected* among a set of candidate planning solutions, as the one that minimizes the worst-case adaptation cost. To the best of our knowledge, references Zhao et al. (2009, 2011); Maghouli et al. (2011) are the only applications that consider adaptation cost in the power system literature. Real option valuation has been also used to obtain flexible planning decisions Blanco et al. (2011). Related concepts of flexibility are presented in Balijepalli and Khaparde (2010).

In this chapter, we focus on finding flexible planning solutions. However, rather than selecting the best candidate, we propose an optimization model that actually *designs* the flexible system. The model balances both the adaptation and investment costs; thus, the decision maker can adjust, according to its preferences, the degree of flexibility in the final design. Our model uses scenarios only to guide the flexible investment directions, and therefore the solution —unlike the work Zhao et al. (2009)— is significantly less sensitive to the choice of scenario.

However, adaptation between scenarios is not meaningful when uncertainty characterizing the scenarios is small. Rather, we use adaptation processes when the attributes of the revealed scenario significantly differ from the nominal one. In cases where scenario impacts over the planning solution are not significant, robustness to these small impacts could be more valuable than flexibility. Uncertainties can be classified into different categories according to their nature as random and nonrandom as exposed in Maghouli et al. (2011); Buygi et al. (2006). But, for the purposes of flexible planning, we classify them according to the impact produced on the solution; where impact can be defined in terms of change in objective function and direction of optimality of the perturbed solution with respect to the benchmark case.

According to the impact level, we distinguish two types of uncertainty:

1. *global* as those that produce a significant different trend in the solution. Examples of global uncertainties can be the implementation of emissions policies, important shifts in demand, unavailability of a resource such as coal or natural gas, regulation regarding nuclear power plants operation or, an important drop in investment costs, among others. They are typically categorical, i.e., not necessarily a numerical value. A policy-based global uncertainty like imposition of a carbon dioxide (CO_2) cap can have two realizations: "yes" or "no."

2. *local*, attached to each global uncertainty, are used to represent the imperfect knowledge of the global through uncertainty sets. If the global uncertainty realization is categorical, its corresponding local values would represent the range of values in cases where it applies. For example, in the case of a CO_2 cap, the local uncertainty only appears when the carbon cap (CO_2^{cap}) is actually imposed, and it would represent a range of possible values of the cap.

Fig. 6.1 shows the case of a numerical global uncertainty surrounded by its local uncertainty set that grows in time.

Based on this uncertainty classification, we define scenarios for flexibility analysis constructed by realizations of global uncertainties. However, there is as yet no power system application reported in the literature that actually allows imperfect knowledge of scenarios. If an scenario considers low natural gas price being $4/MMBTU, and another considers high natural gas price being $8/MMBTU, solutions still can be sensitive for values surrounding these deterministic numbers. So, is there any reason why a high natural gas price assumption could not be either $7.5/MMBTU or $8.9/MMBTU? We believe the answer is no. This is what motivates consideration of local uncertainties where the attributes of each scenario become fully parameterized through local uncertainty sets.

Traditionally, decision-making problems under uncertainty have been addressed by either probabilistic methods like SP, decision analysis tools like minimization of regrets, and more recently, by RO techniques. Probabilistic methods require distributions of random data, which are not always easy to obtain. These methods based on distributions are insufficient by themselves under presence of nonrandom uncertainties such as policies, preferences, and government decisions, which cannot be modeled by of distributions Kouvelis and Yu (1997). Mostly, regret minimization employs multiple scenarios created by nonrandom uncertainties. However, the solution is also sensitive to the choice of scenarios Higle and Wallace (2002). The decision maker chooses the best solution as that which shows the most similar performance to the best solution of each scenario Kouvelis and Yu (1997). In probabilistic methods decisions are made before the scenario occurs; whereas in decision theory (or risk analysis) tools, decisions are made based on consequences of scenario occurrence Miranda and Proenca (1998a). This also applies to minimization of adaptation cost where consequences of wrong decisions are actually evaluated. Some applications of regrets minimization in power system planning are reported in references Maghouli et al. (2011); Gorenstin et al. (1993); Miranda and Proenca (1998a); De la Torre et al. (1999); Fang and Hill (2003); Cámac et al. (2010); Arroyo et al. (2010).

Appart from SP and regret minimization, RO has become a popular and powerful tool for handling uncertain-but-bounded data in optimization problems Ben-Tal et al. (2004, 2009). Obtaining data bounds from historical information is easier than determining the probability distributions. The robust counterpart of a RO model is tractable as long as uncertainty sets are also tractable, which is usually the case. Recent applications of RO in power systems are in plug-in hybrid vehicles Hajimiragha et al. (2011), security constrained unit commitment Street et al. (2011), and unit commitment under wind output uncertainty Jiang et al. (2012).

This chapter, rather than chooosing the most flexible plan among a set of candidate solutions, designs a flexible system. A reduced-but-representative set of candidates is used to train the model, whereas imperfect knowledge of each scenario is modeled with the corresponding local uncertainties. The resulting "doubly" uncertain optimization problem is tackled via

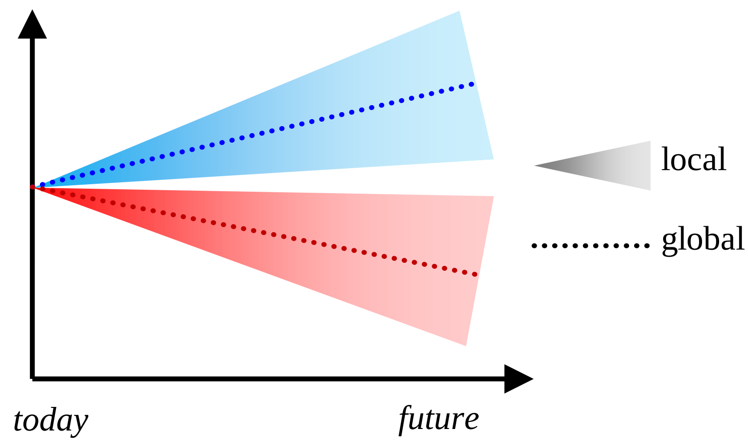


Figure 6.1: Global and local uncertainties

ARO. In this approach, we minimize, not only investment and operational cost, but also the future adaptation cost of the system to any other scenario's conditions. A double Monte Carlo method is performed to verify both the adaptability of the system to new environments and also the robustness against local perturbances in data.

6.3 A Capacity expansion planning Model

The CEP problem consists of, in general, identifying the most cost-efficient energy portfolio that will supply the energy needs of the system in a sustainable and resilient way. "Identifying" refers to finding the right amounts on investments in time and location such that future energy needs are satisfied by considering technical, societal and environmental issues, and uncertainty.

The analytical version of the CEP problem used in this work is stated as deciding how much power capacity to invest in from a set of fossil fuel and renewable generation technologies. Finding the best portfolio not only requires minimizing costs and satisfying demand, but also variability caused by renewable generation and different sources of uncertainty in costs, prices, regulation, policies, and demand. We model a multi-stage long-term investment problem that receives technical and economic signals from the annual operating problem based on a LDC and a DCOPF.

6.3.1 Uncertain Planning

In this section, we present a complete version of the CEP problem accounting for uncertainty. Global uncertainties are bolded, and local uncertainties have a tilde $\sim$. The set that contains all the scenarios is Θ. The CEP problem under one scenario $\boldsymbol{\omega} \in \Theta$ can be stated as:

$$\begin{aligned}
\underset{Cap^{\text{add}}, P, \delta}{\text{minimize}} \quad & \sum_{t,i,j} \zeta^{t-1} \tilde{\mathbf{I}}_{i,j,t,\boldsymbol{\omega}} Cap^{\text{add}}_{i,j,t,\boldsymbol{\omega}} \\
& + \sum_{t,i,j} \zeta^{t-1} \left(OM^{\text{f}}_{j,t} Cap_{i,j,t,\boldsymbol{\omega}} + OM^{\text{v}}_{j,t} \sum_{s} P_{i,j,s,t,\boldsymbol{\omega}} h_s \right) \\
& + \sum_{t,i,f,s} \zeta^{t-1} \widetilde{\mathbf{FC}}_{i,f,t,\boldsymbol{\omega}} \left(\sum_{m \in \Psi_f} H_m P_{i,m,s,t,\boldsymbol{\omega}} h_s \right) - \zeta^{T-1} \sum_{i,j} \widetilde{\mathbf{SV}}_{i,j,\boldsymbol{\omega}} Cap_{i,j,T,\boldsymbol{\omega}}
\end{aligned} \tag{6.1}$$

subject to

$$Cap_{i,j,t,\boldsymbol{\omega}} = Cap_{i,j,t-1,\boldsymbol{\omega}} + Cap^{\text{add}}_{i,j,t,\boldsymbol{\omega}} - Cap^{\text{ret}}_{i,j,t}, \ \forall i, j, t \tag{6.2}$$

$$Cap_{i,j,0,\boldsymbol{\omega}} = Cap^{\text{existing}}_{i,j}, \forall i, j \tag{6.3}$$

$$\sum_{i,j} Cap_{i,j,t,\boldsymbol{\omega}} \geq \sum_{i \in \Phi} (1 + r)\, \tilde{\mathbf{d}}_{i,\text{peak},t,\boldsymbol{\omega}}, \ \forall t \tag{6.4}$$

$$P_{i,j,s,t,\boldsymbol{\omega}} \leq \widetilde{CC}_{i,j,s} Cap_{i,j,t,\boldsymbol{\omega}}, \ \forall i, j, s, t \tag{6.5}$$

$$\sum_{s \in \mathcal{S}} P_{i,j,s,t,\boldsymbol{\omega}} h_s \leq \widetilde{CF}_{i,j} Cap_{i,j,t,\boldsymbol{\omega}} \sum_{s \in \mathcal{S}} h_s, \ \forall i, j, t \tag{6.6}$$

$$\sum_{j} P_{i,j,s,t,\boldsymbol{\omega}} - S^{\text{base}} \sum_{k} b'_{i,k} \theta_{k,t,\boldsymbol{\omega}} \geq \tilde{\mathbf{d}}_{i,s,t,\boldsymbol{\omega}}, \ \forall i, s, t \tag{6.7}$$

$$S^{\text{base}} b_l \left| \sum_{i} S_{i,l} \theta_{i,t,\boldsymbol{\omega}} \right| \leq \tilde{\mathbf{F}}^{\max}_{l,t,\boldsymbol{\omega}}, \ \forall l, s, t \tag{6.8}$$

$$\sum_{u,s} P_{i,u,s,t,\boldsymbol{\omega}} h_s \geq \tilde{\rho}_{t,\boldsymbol{\omega}} \sum_{j,s} P_{i,j,s,t,\boldsymbol{\omega}} h_s \tag{6.9}$$

$$\sum_{j,s} E^{\text{CO}_2}_{j} P_{i,j,s,t,\boldsymbol{\omega}} h_s \leq \widetilde{\mathbf{CO}}^{\text{cap}}_{t,\boldsymbol{\omega}} \tag{6.10}$$

$$\sum_{g,s} H_g P_{i,g,s,t,\boldsymbol{\omega}} h_s \leq \tilde{\mathbf{g}}^{\max}_{t,\boldsymbol{\omega}} \tag{6.11}$$

ζ is the discount factor and T is the planning horizon. The objective function (6.1) is composed of the total investment cost caused by the additions of new generating capacity Cap^{add}, the total operating cost which is the sum of the fixed (rent, water use, facility services) and variable operating (depends on actual energy production) cost. Also, the salvage value is maximized to guarantee the installed capacity has a value in the end of the planning horizon. I is the per-MW investment costs of each technology, OM^{f} is the fixed O&M cost, OM^{v} is the variable O&M cost, and FC is the fuel cost for coal, natural gas, and uranium. SV is the salvage value of each unit in the end of the planning horizon and is assumed to be a percentage of the investment cost.

Cap_t, the installed capacity available throughout period t, as shown in (6.2), is continuously updated balancing the capacity investments or additions Cap^{add} and the deterministic retirements of capacity $Cap^{\mathrm{ret}}_{i,j,t}$ starting period t, and the period $t-1$ cumulated capacity $Cap_{i,j,t-1}$. At $t = 0$, capacity is the existing infrastructure at that moment as shown in (6.3).

Total capacity of the system must satisfy reserve margin r with respect to peak demand $\tilde{d}_{i,\mathrm{peak},t}$ as described in eq. (6.4). Power produced by each individual technology, especially renewables, in different periods of a typical day is bounded by the capacity credit CC as in (6.5). With CC, we consider resource (wind speed, solar radiation) reduced availability in each of the defined LDC steps of a typical day. For WND and SUN, this availability is much smaller than for the rest of the units.

Energy production is bounded by the capacity factor in (6.6). CF is the ratio between the average power produced in a specific period and its nominal capacity. Given the variability of renewable resources, both wind and solar CFs are the lowest. h represent the duration of each LDC step in hours.

Total power generation plus (minus) inports (exports) of power, expressed using angular differences, coming into (leaving from) every region must be enough to satisfy demand at every step of the LDC. The demand balance constraint is expressed in terms of the voltage angles of buses that are actually connected to the demand bus in consideration as described in (6.7), with $b'_{i,k} = \sum_l b_l S_{i,l} S_{k,l}$.

The power flowing by each path in the network is bounded by the thermal limits on the transmission lines. If flows are approximated and expressed in terms of angular differences, maximum (and minimum) flow constraints are as shown in (6.8). S represents the network connectivity matrix, b line susceptances in per unit, and S^{base} the base power of the system.

Constraint (6.9) reflects a renewable portfolio standard. It guarantees that at least a percentage ρ of the energy produced must come from renewable resources in the scenarios where it applies. When it does not apply, we set $\rho = 0$. In constraint (6.10), a cap in carbon emissions is established. E^{CO_2} is the amount of CO_2 emissions per unit of energy each technology produces. If the scenario does not consider this cap, $\widetilde{\mathbf{CO}}^{\text{cap}} = +\infty$. Uncertainty in natural gas reserves is modeled in (6.11). It annually limits the use of gas. When it does not apply, $\mathbf{g}^{\max} = +\infty$.

6.4 Flexibility

In the works Zhao et al. (2009) and Zhao et al. (2011), the concept of flexible planning was introduced supported by the idea of adaptation costs. For each scenario, the optimal planning solution is computed and is referred to as a candidate. The flexible system is *chosen* among this set of candidates as the one that minimizes the worst-case adaptation cost.

Although the computation of the flexible system is straightforward, the final solution has some disadvantages. For example, there are some scenarios whose solutions dominate the rest. We mean by dominance a solution that is always chosen as the most flexible given that the cost of any other solution to adapt to the dominant is very high; and the cost of the dominant to adapt to other solution is very low. Also, the solution based on candidates is very sensitive to the selection of scenarios. If the set of scenarios changes, so does the set candidates, and therefore so does the final solution.

Our approach does not choose a solution, but instead, our approach designs it. The flexible system is part of the set of decision variables in the optimization model. Also, we do not obtain dominant solutions since the formulation guides the model to choose a "central" solution. Therefore the sensitivity to the choice of scenarios is much less. Scenarios are only used to train the model, and once a good set of representative scenarios —or cluster— is used, the changes in the solution are minimum.

6.4.1 Flexible planning model —conceptual description

Fig. 6.2 illustrates our concept of finding a flexible solution. Basically, we want to find a trajectory $x_t^f, \forall t$ which is "close" to each of the scenario feasibility sets. The optimal solution

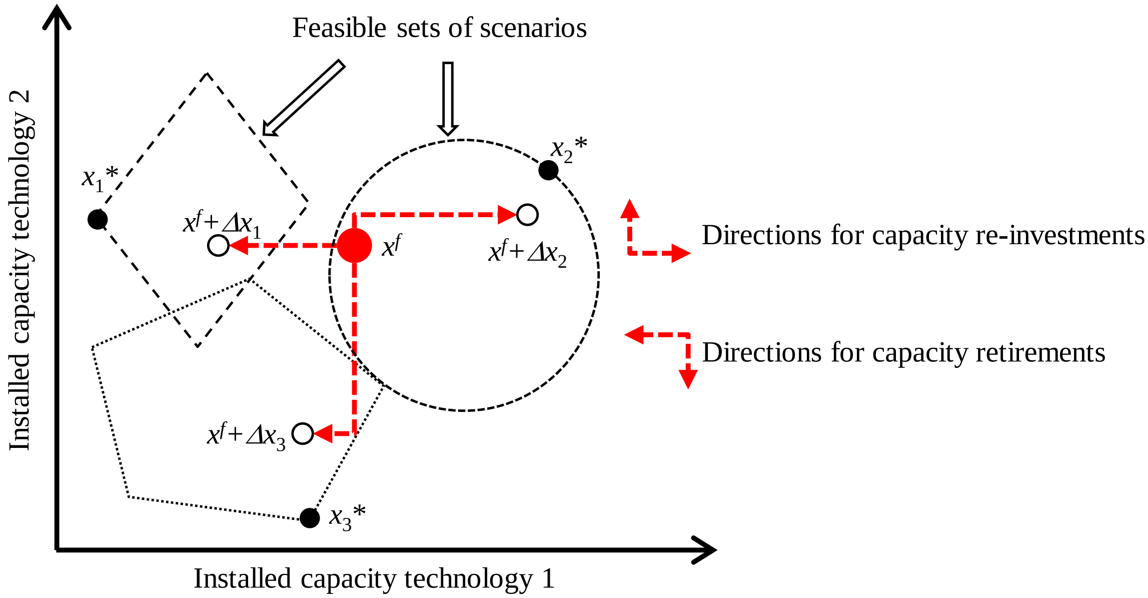


Figure 6.2: Concept of flexible solution

under scenario $\boldsymbol{\omega} \in \Theta$ is depicted as $x^*_{\boldsymbol{\omega}}$. Each of them perform well in the scenario they were designed for, they do not constitute any input for our model.

We assume the existence of a common power system x^f, a core, that can be adapted to any condition. To guarantee adaptation, we force x^f to adapt to a predefined set of scenarios. This consists of adding to or substracting from x^f capacity in the direction $\Delta x_{\boldsymbol{\omega}}$ such that the new adapted power system $x^f + \Delta x_{\boldsymbol{\omega}}$ performs well under the local uncertain conditions of scenario $\boldsymbol{\omega}$. The criterion to achieve flexibility, in addition to the minimization of the investment cost of x^f, consists of the minimization of the AC $\sum_{\boldsymbol{\omega}} I_{\boldsymbol{\omega}} \Delta x_{\boldsymbol{\omega}}$ (that is, the total "distance" weighted by the per-MW investment cost) plus the cost of operating each adapted system. Each of the re-expanded systems do not necessarily coincide with the optimal solutions $x^*_{\boldsymbol{\omega}}$ since the re-investment starts from x^f and not from zero.

6.4.2 Maximizing future flexibility

The resulting optimization problem is as follows:

$$\underset{Cap^{\text{add}},P,\delta}{\text{minimize}} \quad \sum_{t,i,j,\boldsymbol{\omega}} \zeta^{t-1} \tilde{\mathbf{I}}_{i,j,t,\boldsymbol{\omega}} Cap^{\text{add}}_{i,j,t}$$

$$+\sum_{t,i,j,\boldsymbol{\omega}} \zeta^{t-1}\left(OM^{\text{f}}_{j,t}\,Cap^{\boldsymbol{\omega}}_{i,j,t} + OM^{\text{v}}_{j,t}\sum_{s} P_{i,j,s,t,\boldsymbol{\omega}} h_s\right) \tag{6.12}$$

$$+\sum_{t,i,f,s,\boldsymbol{\omega}} \zeta^{t-1}\widetilde{\mathbf{FC}}_{i,f,t,\boldsymbol{\omega}}\left(\sum_{m\in\Psi_f} H_m P_{i,m,s,t,\boldsymbol{\omega}} h_s\right) - \zeta^{T-1}\sum_{i,j,\boldsymbol{\omega}} \widetilde{\mathbf{SV}}_{i,j,\boldsymbol{\omega}}\,Cap^{\boldsymbol{\omega}}_{i,j,T}$$

$$+\beta \underbrace{\sum_{t,i,j,\boldsymbol{\omega}} \zeta^{t-1}\left(\tilde{\mathbf{I}}_{i,j,t,\boldsymbol{\omega}}\Delta Cap^{+}_{i,j,t,\boldsymbol{\omega}} + \tilde{\mathbf{R}}_{i,j,t,\boldsymbol{\omega}}\Delta Cap^{-}_{i,j,t,\boldsymbol{\omega}}\right)}_{\text{Adaptation cost}}$$

subject to:

$$Cap^{f}_{i,j,t} = Cap^{f}_{i,j,t-1} + Cap^{\text{add}}_{i,j,t} - Cap^{\text{ret}}_{i,j,t},\ \forall i,j,t,\boldsymbol{\omega} \tag{6.13}$$

$$Cap^{\boldsymbol{\omega}}_{i,j,t} = Cap^{f}_{i,j,t} + \Delta Cap_{i,j,t,\boldsymbol{\omega}},\ \forall i,j,t,\boldsymbol{\omega} \tag{6.14}$$

And constraints (6.3), (6.4), (6.5), (6.6), (6.7), (6.8), (6.9), (6.10), (6.11) $\forall \boldsymbol{\omega} \in \Theta$

The modified objective function (6.12) represents total investment and operational cost plus a penalized adaptation cost. Cap^f (analogous to x^f) is the installed capacity of the flexible system. Each $\Delta Cap_{\boldsymbol{\omega}}$ (analogous to $\Delta x_{\boldsymbol{\omega}}$) is a real number, therefore the adaptation cost penalizes positive and negative adaptation actions through investment $\tilde{\mathbf{I}}$ and retirement $\tilde{\mathbf{R}}$ cost respectively. The coefficient β is used for controlling the level of flexibility in the solution. Very small values of β produce Cap^f close to zero and large $\Delta Cap_{\boldsymbol{\omega}}$. The reason is that the model prefers not to invest in core capacity but to adapt to any scenario at an apparent low cost. Conversely, large values of β produce large values of Cap^f and $\Delta Cap_{\boldsymbol{\omega}}$ close to zero. In this case, adaptation is apparently too costly and therefore it is better to build core capacity. Although solutions under large β tend to be robust, they are not practical to implement given the significant amounts of investments. Thus, a balance between investment and adaptation cost implies a good selection of β.

Constraint (6.13) represents the updating process of the flexible capacity; whereas constraint (6.14) shows how the system capacity of each scenario is updated. Basically, the flexible system, characterized by Cap^f, grows in different directions determined by $\Delta Cap_{\boldsymbol{\omega}}$ in order to satisfy the conditions of each scenario. The rest of the constraints, from (6.3) to (6.11), specify each year's operational problem under all of the scenarios. This is as if we were running in parallel different power systems —defined by $Cap^{\boldsymbol{\omega}}_{i,j,t}$— to analyze their performance.

6.5 Numerical results

The model proposed is tested on the portfolio investment problem of a 40-year planning five-region US model using 20 2-year periods. The portfolio consists of 13 different technologies including renewables, fossil and alternative fuel production technologies.

Global uncertainties and scenario clustering

Global uncertainties can define an investment pattern throughout the planning horizon. However, detecting when an uncertainty is global may require heuristics and is a process that depends on the particular application. In this work, when a change in an input parameter produces a significantly different technology portfolio from the benchmark, it is considered a global uncertainty.

The benchmark plan is computed assuming data realizations such as Low (L) or High (H) gas price (GP), L or H demand (D) growth rates, L or H wind investment cost (WC), imposition or not of natural gas production limits (GPL), CO_2^{cap}, and Renewable Portafolio Standards (RPS). Although other uncertainties like investment costs of all other technologies, coal and uranium price, capacity credit, and capacity factor are also considered via uncertainty sets, we only describe the parameterization of global and their corresponding local uncertainties.

For those realizations of global uncertainties that can be parameterized by uncertainty sets, we employ three different models to characaterize their growth. In each model, $\chi_{t,real}$ is the central value of the global uncertain variable at year t under realization $real$, χ_{2009} the 2009 ($t = 1$) assumed value, and $\hat{\chi}_{real}$ the variability of χ under realization $real$. Since we consider two realizations for each global uncertainty, $real$ can be either "L" and "H", or "Yes" and "No". μ is the annual change rate in exponential models or slope in the linear model, ξ the variability quantified as a fraction of the central value of the uncertainty, and ϑ the annual rate at which the variability changes. The models are as follows:

1. Exponential growth:

$$\chi_{t,real} = \chi_{2009} \left(1 + \mu_{real}\right)^{t-1}$$

Table 6.1: Local uncertainty parameters

G. Uncertainty	real	Parameters				
		χ_{2009}	μ_{real}	ξ_{real}	ϑ_{real}	κ_{real}
GP	L[1)]	\$3.5/MMBTU	2%	50%	0.1%	N.A.
	H[2)]	\$3.5/MMBTU	3%	50%	-0.8%	7
GPL	Yes[1)]	6.7 tcf[2]	4%	10%	3%	N.A.
	No[1)]	$+\infty$	N.A.	N.A.	N.A.	N.A.
D	L[1)]	725.96 GW	1.3%	5%	4%	N.A.
	H[1)]	725.96 GW	2.2%	3%	4%	N.A.
RPS	Yes[3)]	9.1%	0.54%/y	16.7%	0%	N.A.
	No[3)]	0%	0%/y	0%	0%	N.A.
CO_2^{cap}	Yes[1)]	2,270 MMeTon[3]	-3%	10%	5%	N.A.
	No[1)]	$+\infty$	N.A.	N.A	N.A.	N.A.
WC	L[1)]	\$2.51 Mill/MW	-1.4%	15%	1.3%	N.A.
	H[1)]	\$2.51 Mill/MW	-0.11%	15%	1.3%	N.A.

2. Asymptotic exponential growth:

$$\chi_{t,real} = \chi_{2009}\left[\kappa_{real} - (\kappa_{real} - 1)(1 + \mu_{real})^{1-t}\right]$$

3. Linear:

$$\chi_{t,real} = \chi_{2009} + \mu_{real}(t-1)$$

The variability model in each case is of the form $\hat{\chi}_{t,real} = \xi_{real}(1 + \vartheta_{real})^{t-1}\chi_{t,real}$.

Parameters that determine each realization of global uncertainties are presented in Table 6.1. The superscripted indices attached to each global uncertainty denote the model in which parameters will be used. The local uncertainty model of each global uncertainty χ is $\tilde{\chi}_{t,real} = \chi_{t,real} + \Omega\,\eta_{t,real}\,\hat{\chi}_{t,real},\ \|\eta_{real}\|_1 \leq 1$[1].

After running a deterministic planning model under current power market conditions, a benchmark solution is obtained (see Table 6.2 for details regarding the benchmark assumptions). It has a strong tendency to invest in NGCC units across the country. However, when some input parameters change, one at the time, so does the benchmark portfolio. For instance, if GP is H, NUC power and a little coal power investments replace the NGCC observed in

[1]This model corresponds to the Manhattan uncertainty set presented in Chapter 2
[2]tcf = trillion cubic feet
[3]MMeTon = Million Metric Ton

Table 6.2: Global uncertainty realizations for cases I and II

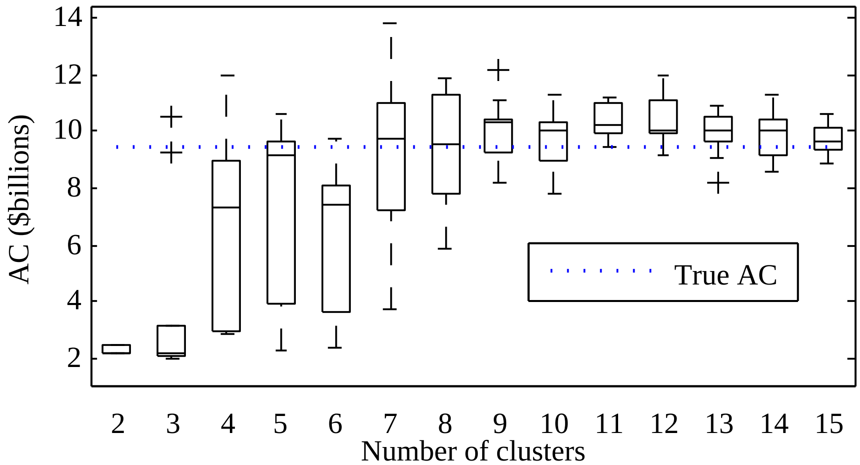
Cluster/scenario	GP	GPL	D	RPS	CO_2^{cap}	WC
Benchmark	L	No	L	No	No	H
1	L	No	L	No	No	L
2	L	No	L	Yes	No	H
3	L	No	L	Yes	Yes	L
4	L	No	H	No	Yes	H
5	L	Yes	L	Yes	Yes	H
6	H	No	L	Yes	Yes	L
7	H	No	H	No	Yes	L
8	H	Yes	L	No	No	L
9	H	Yes	L	Yes	Yes	H
10	H	Yes	H	Yes	No	H

the benchmark. WND is an attractive technology under the scenario of WC being L and when RPS are imposed. If growth rates of D change from L to H, investments increase in the same proportion as the benchmark. When CO_2^{cap} is implemented, WND, NUC, and NGCCCS investments become attractive. Imposition of GL causes some NGCC to be replaced by some NUC and coal-fired units in the central part of the country.

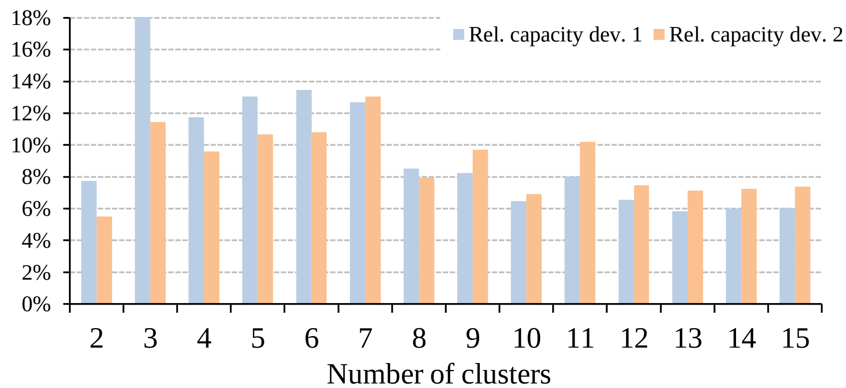
Combining each of the two categorical realizations of the six global uncertainties results in $2^6 = 64$ scenarios[4]. We use the k-medoids clustering method (Kaufman and Rousseeuw (1990)), fed by the 64 individual optimal investment vector solutions, to select the most representative combination of global uncertainty categorical realizations. Every time the clustering tool is run with a fixed number of clusters, given the randomness in its process, it usually returns a different cluster. That is the reason why for a fixed number of clusters, a different solution can be obtained.

Fig. 6.3a shows the average adaptation cost (AC) distributions resulting from running our model of Section 6.4.2 (without local uncertainties) under different number of clusters. The randomness in the clustering tool is the source of uncertainty in the boxplots. The dashed blue line represents the (true) average AC when all the 64 scenarios are incorporated in the model. By looking at Fig. 6.3a, when less than 8 clusters are selected, AC distributions are

[4]Scenario is defined as a list of realizations of the six global uncertainties under consideration; and cluster is a list of representative scenarios



(a) Distributions of per-scenario adaptation costs



(b) Average relative installed capacity deviation

Figure 6.3: Model behavior under different number of clusters

Table 6.3: Computational features according to cluster sizes

Cluster size	Problem size		DDP performance		
	Variables	Constraints	Iter.	Rel. gap	CPU time
25	218,800	348,120	34	9.5×10^{-6}	23 min
45	384,400	556,220	24	6.2×10^{-6}	34 min
64	555,760	889,080	39	7.2×10^{-6}	118 min

quite far from the blue line; but, as long as this number increases, the AC distribution (and its corresponding optimal portfolios) converges to the true AC. To measure convergence, we computed the average relative installed capacity deviation w.r.t. to the true portfolio and to the average portfolio in each case. These quantities are referred to as "Rel. capacity dev. 1" and "Rel. capacity dev. 2" respectively in Fig. 6.3b. These indicators measure how similar —in average— the solutions of each cluster are. Again, the larger the number of clusters, the closer to the true solution.

Also, we explored the behavior of our model when the number of clusters is high. Each of the cases presented in Table 6.3 was run using the DDP decomposition method presented in 5.4. The ten-cluster case has 140,120 constraints and 89,200 variables. According to Table 6.3, the size of the reduced model (10 clusters) is only 2.5% the size of the complete model, and the reduced model is solved directly by Mosek in approximately 2 minutes. Thus, working with a low number of clusters significantly reduces the computational effort.

An advantage of our flexibility model is that it can can find solutions that are very close to the true optimal portfolio with a reduced number of clusters. With only ten clusters, the relative deviations in capacity are less than 6.5%. Based on our experiments, additional clusters are useful for polishing the solution but do not constitute a major change in the composition of the optimal flexible portfolio.

Next, we present the design of energy portfolios via different approches. Case I illustrates results obtained with the proposed model without local uncertainties, the focus is on balance between robustness and flexibility through parameter β. In case II, the "discrete" flexible plan of Zhao et al. (2009) is adapted to our capacity expansion planning problem. And case III shows results of our proposed model considering local uncertainties and solved by ARO. In

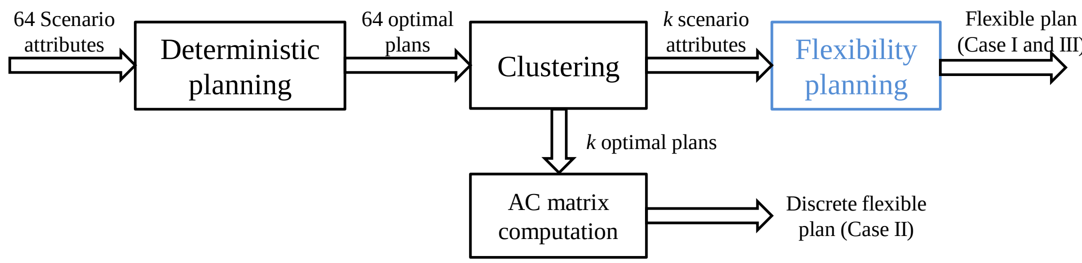


Figure 6.4: Tools interaction

order to clarify how these tools interact, Fig. 6.4 shows the paths taken to obtain the results of each case.

Since a cluster is nothing but a representative subset of the entire scenario set, from now on, we use the words "cluster" or "scenario" to mean the same.

6.5.1 Case I: Flexibility model with perfect knowledge of scenarios

Each of the scenarios and their attributes are presented in Table 6.2. Not only did we choose to work with 10 scenarios due to the reduced computational effort, but also due to the accurate solution they produce. First at all, we focus on balancing adaptation and investment cost (see "Av AC" and "Av IC" respectively). Fig. 6.5 shows the tradeoff between adaptation and investment costs caused by changes in β. For small β there are no investments at all since adaptation is apparently cheap. The adaptation cost plotted represents the average cost needed to adapt to one scenario at some point in time. Large values of β lead to more robust systems where little adaptation is needed. Robustness is achieved by having significant installed capacity of NGCC, NUC, and WND. This flexible system changes smoother with β as depicted in Fig. 6.5.

Total system investments and capacity are plotted in Figs. 6.6a and 6.6b respectively. $\beta = 0.6$ generates a flexible system whose installed capacity is 1,694 GW, close to the maxium peak demand modeled in 2049, which would be above 1,600 GW. The flexible portfolio suggests to invest in NUC (mostly in the West and South-East), WND (in the central part of the country), and natural gas (NGCC and ACT) units (in the east coast). Investments in NGCC units are necessary to replace the retirements by 2033 assumed in our data[5]. In summary,

[5] Capacity retirements are assumed based on the units lifetime and time of operation

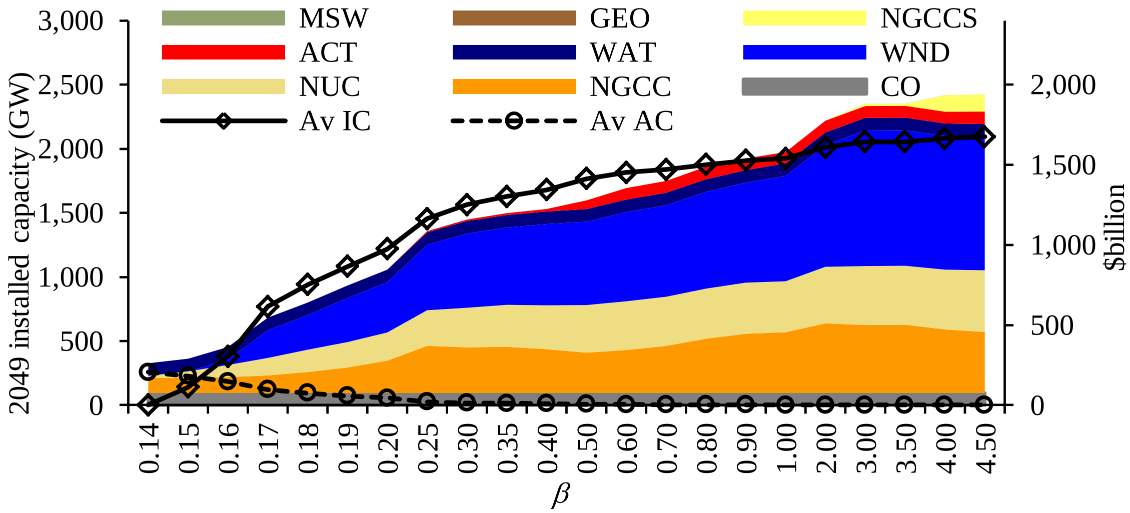
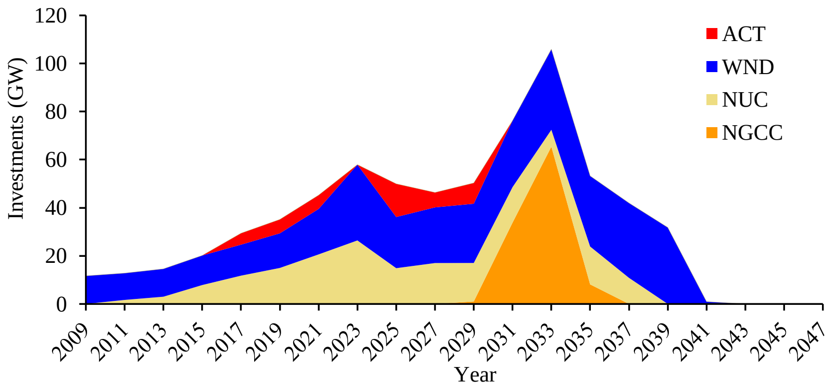
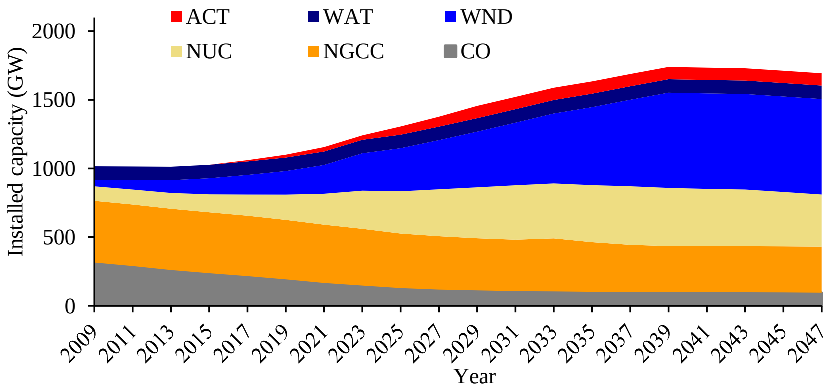


Figure 6.5: Flexible system capacity and costs tradeoff



(a) Case I flexible investments



(b) Case I flexible system capacity

Figure 6.6: Case I optimal portfolio for $\beta = 0.6$

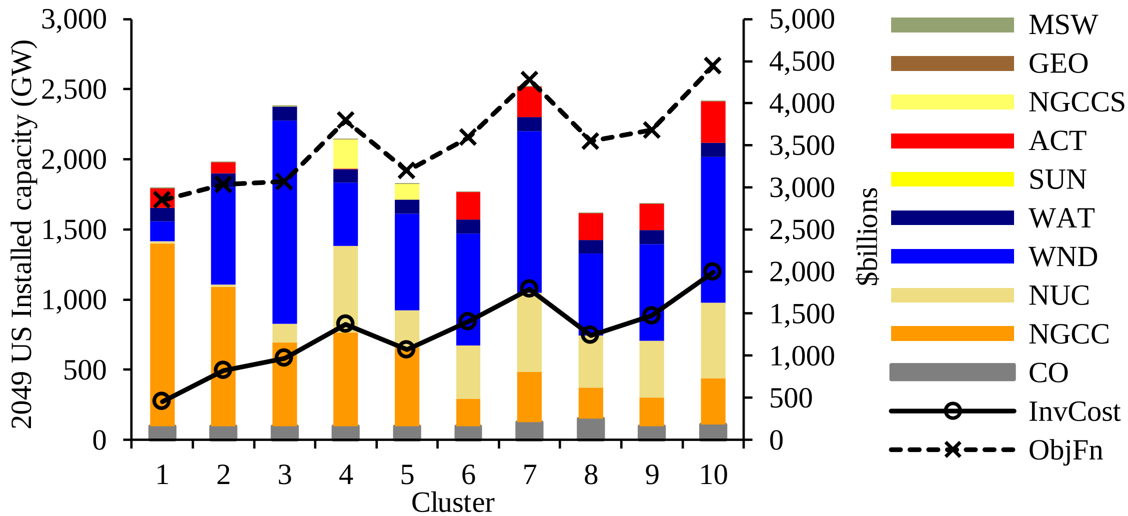


Figure 6.7: Final installed capacity of selected clusters

constructing a flexible system implies keeping quite constant NGCC, increasing NUC, WND, and combustion turbines capacity; and retiring coal. The resulting system is robust to any scenario where demand is low and adaptable to those with high demand. The flexible system invests up to year 2040, after that year it is better off to "see" the realized scenario and adapt the system to it.

6.5.2 Case II: The flexibility approach of Zhao et al. (2009)

Fig. 6.7 shows the ten representative portfolios resulting of individually optimizing the system assuming the occurrence of each of the ten selected scenarios of Table 6.2. Scenarios (1 and 2) with low GP; and without GPL and CO_2 regulation notably favor the investment in NGCC; however, the combination of carbon caps and low gas price is interesting for installing NGCC with carbon sequestration capacity. Imposition of carbon emission caps and RPS support the construction of WND power. These portfolios are the candidates we use for determining the flexible system according to the method exposed in reference Zhao et al. (2009).

The objective function is computed for different values of β analogously to our objective function (6.12). Actually we use eq. (6.15):

$$f\left(x_{\boldsymbol{\nu}}, \beta\right) = \sum_{\boldsymbol{\omega} \in \Omega} \left(IC_{\boldsymbol{\omega}}\left(x_{\boldsymbol{\nu}}\right) + OM_{\boldsymbol{\omega}}\left(x_{\boldsymbol{\nu} \to \boldsymbol{\omega}}\right)\right) + \beta \sum_{\boldsymbol{\omega} \in \Omega} AC_{\boldsymbol{\omega}}\left(x_{\boldsymbol{v} \to \boldsymbol{\omega}}\right) \tag{6.15}$$

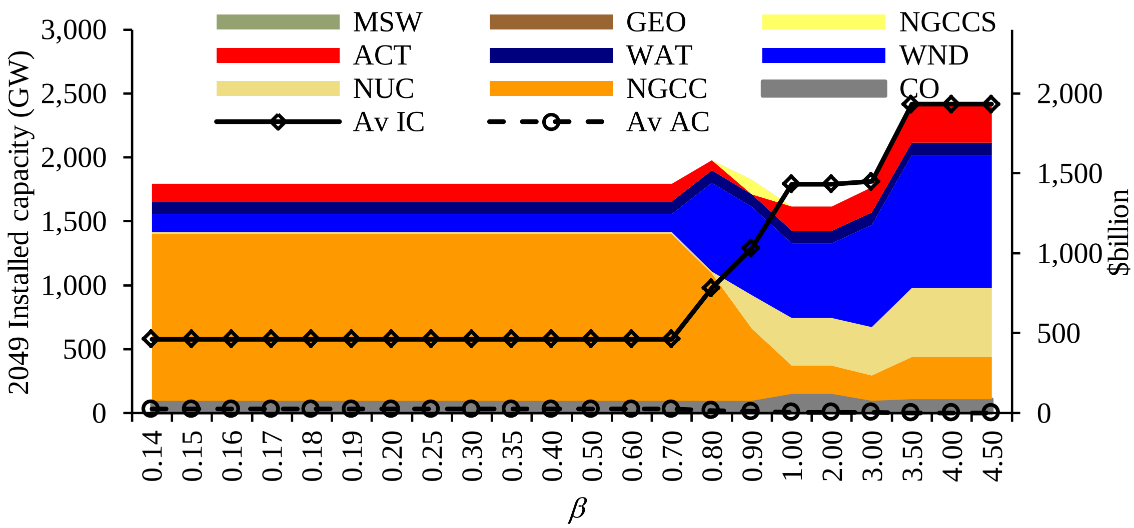
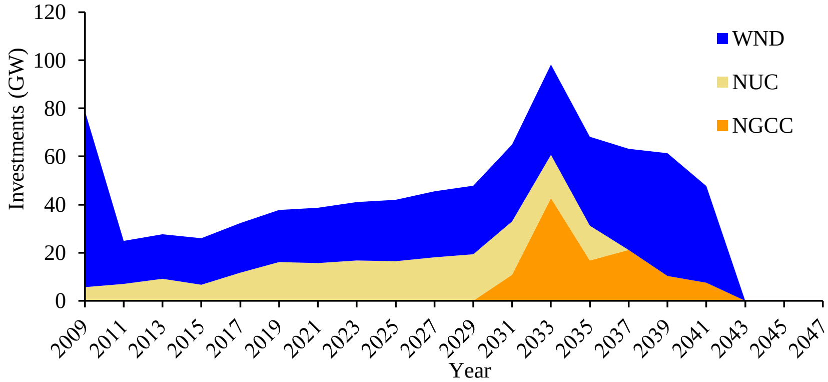


Figure 6.8: Optimal discrete portfolio with respect to β

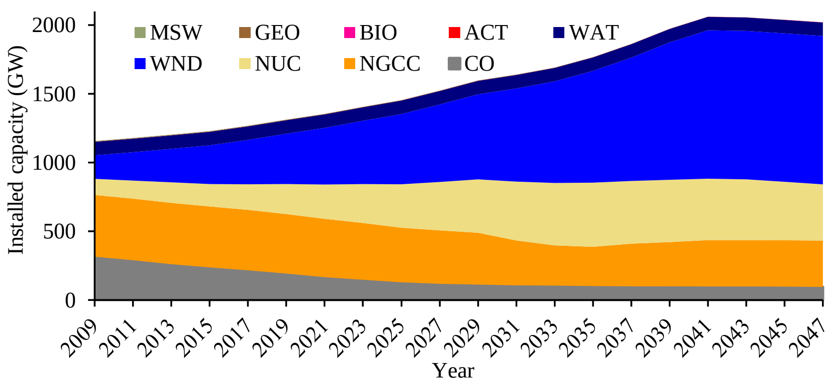
The adapted solution $x_{\boldsymbol{\nu}\rightarrow\boldsymbol{\omega}}$ is obtained by adapting the optimal portfolio $x_{\boldsymbol{\nu}}, \boldsymbol{\nu} \in \Theta$ to each other scenario $\boldsymbol{\omega}$ using an adaptation optimization model. This model optimizes the strategies of adapting a candidate to an scenario, and provides the re-investment decisions needed to efficiently perform under the new scenario by minimizing both the adaptation $AC(x)$ and operational $OM(x)$ costs. The investment cost $IC(x)$ of each candidate x_ν is computed under all of the realizations of investment cost of all scenarios. Then, the flexible system among the candidates is selected as[6]:

$$f^*(\beta) = \min_{\boldsymbol{\nu}=1,\ldots,10} \{f(x_{\boldsymbol{\nu}}, \beta)\}$$

Varying β we "jump" among the candidates to select the best that minimizes $f(x_{\boldsymbol{\nu}}, \beta)$. Fig. 6.8 shows the optimal portfolios as a function of β, which are nothing but an appropriate selection among the ten candidates. Even small changes in β lead to choose a portfolio that was designed under completely different data assumptions. In other words, solution is discontinuous in β.



(a) Case III flexible investments: Cap^{add}



(b) Case III flexible capacity: x^f

Figure 6.9: Case III optimal portfolio

6.5.3 Case III: Flexible planning under imperfect knowledge of scenarios

The scenarios selected to train the model were selected as explained previously. We observed that with a ten-scenario cluster the flexible solution shows little statistical variability. To reduce the conservatism level, linear decision rules for voltage angles as function of demand are implemented. Refer to Chapters 4 and 5 for additional details in adjustable robust optimization.

Since this design is robust to all local uncertainties, it is larger in size than the flexible system of case I as shown in Figs. 6.9a and 6.9b. The portfolio show was obtained setting $\beta = 0.35$. This system invests more in WND in the west coast. The reason is that it is exposed to more economic risk, especially in gas price, due to the modeling of local uncertainties, whereas the model of case I is not. WND and NUC power seems to absorb risks of local uncertainties of the global scenarios such as CO_2^{cap}, GPL, RPS, and GP. Similar to the behavior in Case I, investments are observed until 2042. After that year, it is more optimal to adapt to different scenarios, if necessary, than to install more capacity.

A key advantage of this model compared to cases I and II is the satisfactory performance under realistic conditions. To see what the key difference between the systems of case I and III is, we simulate the actual evolution of the power sector where the decision maker updates the investment decisions by using a folding horizon approach.

6.5.4 Folding horizon simulation

Fig. 6.10 is a flow diagram of one iteration of the folding horizon simulation. In each iteration, a 40-year trajectory of global uncertainty realizations is obtained via two-state discrete time Markov chains. To decide whether the flexible designed needs adaptation to the circumstances proposed by the Markov chain or not, a robustness test is performed by simulating multiple times a production cost model (plus constraints that depend on the global uncertainty realizations) using random data generated from realizations of local uncertainties. This test can help to potentially reduce the cost of the final plan since adaptation might be not necessary when the design succeds the test, which is more likely in the ARO-based design. Success or

[6] The work Zhao et al. (2009) actually minimizes the worst-case adaptation cost rather than the penalized total cost. However, for our purposes, such a difference does not affect any of our conclusions

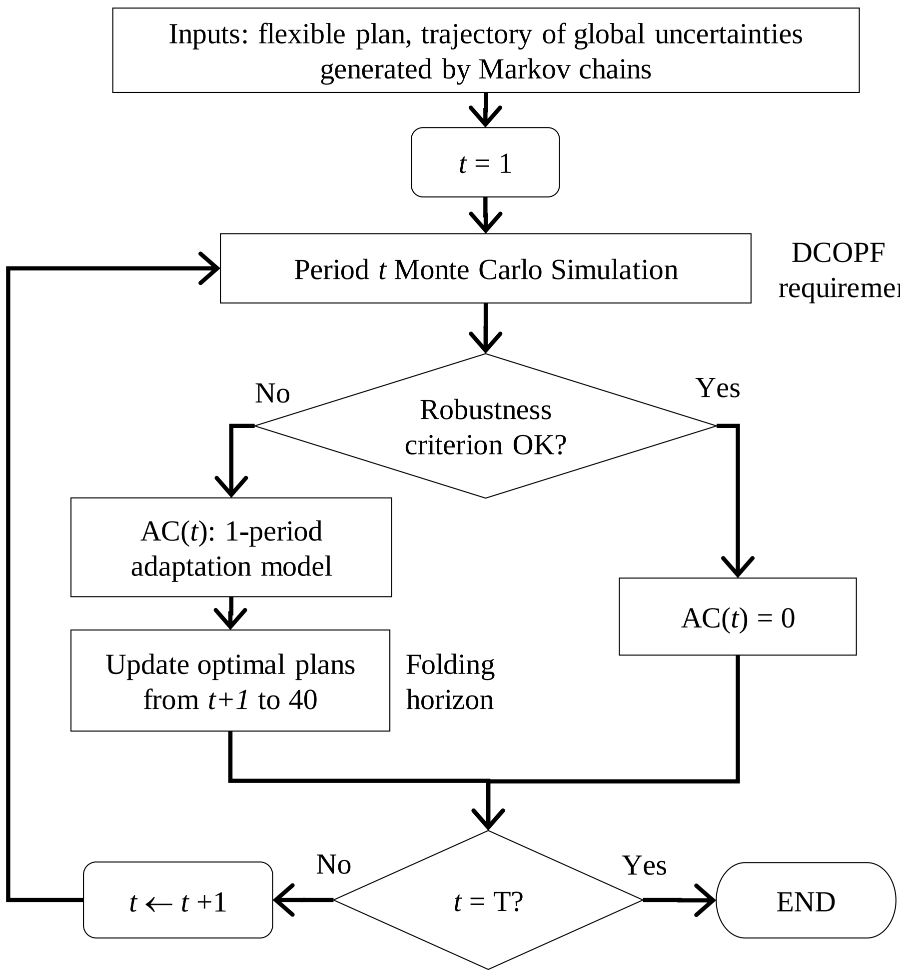


Figure 6.10: One iteration of the folding horizon simulation

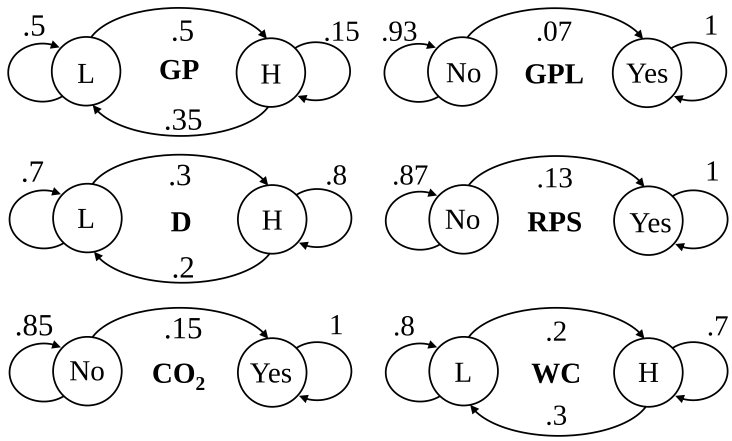


Figure 6.11: Markov chains

failure is determined by the EENSP, which we define as the expected ratio between the total energy not served and realized energy demand. Only when the test is not passed, a single period planning model is run to determine both re-invesments needed and its corresponding (adaptation) cost. Since re-investments decisions are actually implemented, the initial flexible plan has partially changed and therefore needs to be updated using the proposed flexibility model. This recursive updating process is known as folding horizon. The system is exposed to different Markov chain trajectories so as to get stable statistics related to the adaptation cost.

Markov chains are used to model the evolution of each global uncertainty. They are considered to be independent, although that is not necessarily the case. We model six two-state discrete time Markov chains as illustrated in Fig. 6.11. Each number in the plot represents the transition probability among states. Those correspondng to GPL, RPS and CO_2^{cap} global uncertainties are modeled with an absorbing state. When the policy is actually implemented, it is assumed to hold up to the planning horizon, at least. Each of these three chains is started at the “No” state, and according to its transition model, after some time the “Yes” state is achieved. This (hitting) time is also a random variable resulting in a geometric distribution; and as an statistical fact, the expected time to get to the absorbing state is the inverse of the probability of implementing the policy. For instance, the assumed expected time to implement the RPS policy is $1/0.133 = 7.5$ periods (15 years.)

A summary of the simulation results is presented in Table 6.4. In here, the average annual

Table 6.4: Folding horizon simulation results

Model	Simulation		Optimization	
	IC($bill)	AC($bill/y)	IC($bill)	AC($bill/y)
Case III	1,789.9 (88.7)	11.3 (6.0)	2,020.2 (49.9)	16.4 (26.2)
Case I	678.6 (107.8)	36.2 (7.2)	1,452.2 (46.5)	4.1 (6.5)

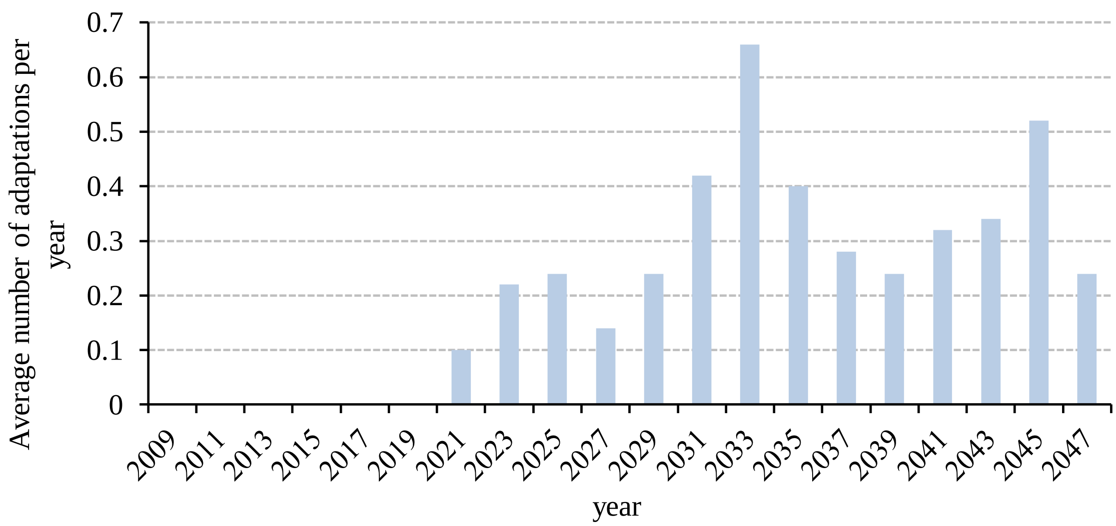


Figure 6.12: Average yearly adaptations of ARO based designs

adaptation costs is averaged over every simulated trajectory of uncertainties. On the other hand, as expected, the mean adaptation cost of the ARO plan (case III) is lower than the mean adaptation cost of the initial design —see AC under label "Optimization" in Table 6.4— given the good performance of the system. Unlike the ARO based plan, the Deterministic model (case I) performs poorly under every chain trajectory. That is the reason why the adaptation cost ends up so high compared to the obtained in the initial optimization. AC in the simulation are such that the EENSP is lower than a threshold value (see Fig. 6.10). Fig. 6.12 shows the average number of adaptations per year performed by the ARO based model. In the worst-case situation, 0.66 adaptations per year were performed; however, during the first 11 years the initial designed system performed well in any of the 50 trajectories. By taking the average over the trajectories, we observed 4.4 adaptations in 40 years (or 0.11 adaptations every year), which is approximately an adaptation every 9.2 years. Since the per-year cost of each adaptation is \$11.3 billions in average (see Table 6.4), it roughly means that \$103.7 billions would be needed every 9 years for correcting initial planning decisions. If lower adaptation cost is desired —although higher investment cost, a larger β can be selected in the optimization process.

Although the investment cost of the deterministic model is significantly lower than the ARO model, it has to employ excessive adaptation processes to meet the robustness EENSP threshold. This recurrent adaptation is not convenient given timing constraints, and re-investment decisions usually have to be implemented quicker than those made long time in advance. These adavantages of the RO model compared to the deterministic are the reasons why the two-level model of uncertainty should be considered in long-term studies.

6.6 Conclusions

We present a novel methodology that obtains robust and flexible future capacity expansion plans under diverse types and sources of uncertainty. Uncertainties are classified as global and local according to the impact. Global uncertainties allow us to both create scenarios and train the flexibility-based planning model; whereas local uncertainties allow us to both model the level of imperfect knowledge within each scenario and create uncertainty sets. Our methodology, rather than chooosing the most flexible plan among a set of candidate solutions, actually designs a flexible system by minimizing its future adaptation cost to any other scenario's conditions. This reduces significantly the dependence on the choice of scenario. The decision maker can balance the investment and adaptation cost by properly selecting a parameter that controls the degree of flexibility and global robustness of the solution. Results show that a flexible system, i.e., adaptable to future big-impact uncertainties like drastic changes in gas price, carbon regulation, and renewable portfolio standards, is achievable at a reasonable low investment costs with a low number of re-investment decisions. A folding horizon process where global uncertainties are guided by stochastic processes was performed to measure more realistically the degree of flexibility of the system and its cost. By modeling local uncertainties via ARO, the evolution of the system can be even less costly than what is estimated before the actual system operation.

CHAPTER 7. CONTRIBUTIONS AND FUTURE WORK

Traditionally, planning tools have been dedicated valuable efforts to model the physical and economic phenomena involved in power systems. However, given its size and interdependencies with multiple sectors, the evolution of power systems is notably sensitive to multiple sources of uncertainties. The world systems are dynamic; the economy of the country affects the power system economics; policy controls regulations; the environment and its relationship with the power sector is currently a topic of debate; reserves of fossil fuel are not infinite; green power resources are highly volatile; and energy demand keeps growing. All of these issues motivated that this work has focused on studying and proposing different generating capacity expansion planning tools for modeling a more robust and flexible evolution of the power system.

7.1 Contributions of this work

- **Classification of uncertainties as global and local**

  Independently of their nature —random or nonrandom, uncertainties were classified as global and local according to the impact. This partition was found more adequate to address robustness and flexibility in the generation expansion planning. Global uncertainties allowed us to both create scenarios and train the flexibility-based planning model; whereas local uncertainties allowed both modeling the level of imperfect knowledge within each scenario and creating uncertainty sets. Results presented in Chapters 3–5 focus on the modeling of uncertainty sets within a specific collection of global uncertainty realizations. Chapter 6 addresses the modeling of local within global uncertainties driven by the concept of flexibility.

- **Model of imperfect knowledge of scenarios**

Traditionally, uncertainty–related tools assume perfect knowledge of scenarios. Indeed, scenarios are formed by sets by a finite number of random *samples* of each uncertainty. However, a drawback of this assumption relies on the sensitivity of the solution with respect to the sampling process. This in turn generates another uncertainty. In Chapter 6 we have shown that by assuming the decision maker does not know each of the scenario attributes perfectly, he/she can make use of uncertainty sets formed by local uncertainties proper of each specific scenario.

- **Robust optimization for power system capacity expansion planning**

  Robust optimization, a state–of–the–art methodology that efficiently models *multiple* uncertainties by means of uncertainty sets, was introduced for solving capacity expansion problems. RO can be used to model either random or nonrandom uncertainties; however, we concluded that uncertainty sets are better suited for modeling of local uncertainties or imperfect knowledge in high-impact uncertainties. The facts that probability distributions of data —hard to obtain— and that only bounds of uncertain data are needed, and the computational tractability, show that RO is a promising *complementary* tool for power system planning.

- **Affinely adjustable robust optimization in power system capacity expansion planning**

  An uncertain version of the capacity expansion problem where all of the decision variables were parameterized as affine functions of uncertain data was proposed. This model is known as affinely adjustable robust optimization. Actual investments depending on fuel prices and analysis variables (power generation, voltage angles) depending on peak demand were capable of reducing the price or robustness usually faced by standard/static RO. One of the key advantages of this approach is that it is not necessary to explicitly represent scenario trees given the continuous model of the uncertainties. Since this is an improvement of RO for multi-stage optimization, it "enjoys" the same features of RO.

- **Dual dynamic programming for solving adjustable robust optimization**

  Like any tool that models uncertainty is computationally intensive, so is the capacity expansion planning problem when multiple uncertainties and adjustable investment decisions are implemented. Also, as long as the planning horizon increases, so does the problem size. It is important to decompose the optimization model into several pieces in an appropriate way. The uncertain planning model shown in Chapter 4 has been transformed in Chapter 5 to make it suitable and solvable through DDP, a Benders-like decomposition method in which a continuous forward and backward optimization is performed in each iteration. Chapter 6 also shows some results obtained with the DDP algorithm.

- **Robust and flexible planning**

  When it comes to consider global uncertainties in planning, it is expensive constructing a robust infrastructure that can absorb the entire risk. This motivated our search for alternative infrastructures that are flexible under global uncertainties. We proposed a model that designs —rather than choosing— a flexible system by minimizing the future adaptation cost to any other scenario's conditions; and at the same time the model reduces significantly the dependence on the choice of scenario. In our results, it was shown that the models that actually consider the model of local uncertainty within scenarios are more flexible than those that do not.

7.2 Future work

Based on the findings of this work, some aspects that lead to improve these efforts as well as future work on power systems under uncertainties are summarized next:

- **Energy and transportation infrastructure**

  The uncertainty modeling techniques developed in this work can be aimed to expansion models not only of the power sector, but also of the energy and transportation sectors as it has been done in Quelhas (2006); Gil (2007); Ibanez (2011). All of these efforts would require more data and significant modeling transformation to add the concepts developed in this work.

- **Data improvement**

  The studies shown throughout this work can be expanded to a larger version of the US system. However, more disaggregated data are needed to accurately model the geographical interdependencies.

- **Transmission planning**

  Probably, the evolution of the US power system would be more optimal if transmission expansion was cooptimized with generation expansion. This cooptimization along with modeling of any type of uncertainty can also provide flexible transmission corridors that can be extended (adapted) in the future if necessary.

Next bullets are of particular interest and may constitute part of my future research activities:

- **Refinement of renewables variability models**

  Although renewables variability has been addressed in the expansion model, a better modeling of wind, solar and hydro outputs can provide more refined planning solutions. Since these technologies follow weather patterns, there exists historical data that allows improving their corresponding models in the DCOPF.

- **Hydrothermal coordination**

  The hydrothermal coordination faces the uncertainty of water inflows, and it has been usually addressed by scenario analysis. However, with the modeling tools presented in this work, the problem can be solved more efficiently. And confidence intervals based on historical information of water inflows can be used to obtain less risky generation schedules, specially designed for drought seasons.

- **Security constrained optimal power flow**

  Contingencies in OPF have been traditionally modeled explicitly, i.e., the security-constrained OPF models create an imaginary system under the presence of the contingency. However, this is computationally expensive in the case of large power systems; even more in the general $n-k$ case. Through robust optimization methods it might be possible to write an optimization problem with k element outages. A first attempt to this approach in unit commitment problems is presented by Street et al. (2011).

- **Optimal bidding in power markets**

  Price–taker power producers daily play in the day-ahead power market by submitting price and quantity offers. However, they do not know in advance their opponents' strategies; this imperfect information generates a price uncertainty which in turn will affect their profits. Although this is more a game theory problem, robust optimization can be used to create price–quantity offers by reducing the risk due to price volatility. Initial findings on this topic are presented in Baringo and Conejo (2011).

APPENDIX A. TRANSFORMING THE AARC INTO A DDP PROBLEM

In this section, we develop a lemma and a corollary that will help us to obtain a DDP model of the AARC.

Lemma 1. *Let $f_0 : \mathcal{R}^n \mapsto \mathcal{R}$ a convex function, T the planning horizon, D_τ an $m_\tau \times n$ instance matrix of the set of matrices $\{D_1, D_2, \ldots, D_T\}$, x the decision vector. Let the following convex optimization problem:*

$$\begin{aligned} &\underset{x\in\mathcal{R}^n}{\textit{minimize}} && f_0\left(x_1,\ldots,x_T\right) \\ &\textit{subject to} && a_t^\top x_t + \max\left\{\left[D_1x_1;\ldots;D_tx_t\right]^\top\right\} \le b_t, \\ & && \forall t = 1,\ldots,T \end{aligned} \tag{A.1}$$

then, optimizing (A.1) is equivalent to optimizing

$$\begin{aligned} &\underset{x_t\in\mathcal{R}^n, y\in\mathcal{R}^T, v\in\mathcal{R}_+^{T-1}}{\textit{minimize}} && f_0\left(x_1,\ldots,x_T\right) \\ &\textit{subject to} && a_t^\top x_t + y_t \le b_t,\ \forall t = 1,\ldots,T \\ & && D_tx_t \le y_t,\ \forall t = 1,\ldots,T \\ & && y_t - y_{t-1} - v_t = 0,\ \forall t = 2,\ldots,T \end{aligned} \tag{A.2}$$

Proof. The product $D_\tau x_\tau$ is an m_τ column vector. Then, operator $\max(\cdot)$ is taking the maximum out of an array whose dimension is $\sum_{\tau=1}^{t} m_\tau$. Let y_1 be

$$y_1 = \max\left\{\left(D_1x_1\right)^\top\right\}$$

and let y_2 be

$$y_2 = \max\left\{[D_1x_1;\ D_2x_2]^\top\right\}$$
$$= \max\left\{\left[y_1,\ (D_2x_2)^\top\right]\right\}$$

If we move forward applying this idea up to time t, we find that:

$$y_t = \max\left\{\left[y_{t-1},\ (D_tx_t)^\top\right]\right\}$$

which is exactly the second term in the LHS in the constraint of problem (A.1). At every stage use the fact that $\max(a,b) \geq a$ and $\max(a,b) \geq b$ to obtain

$$y_\tau \geq y_{\tau-1}, \text{ and } y_\tau \geq D_\tau x_\tau,\ \forall\tau = 2,\ldots,t$$

Now, problem (A.1) becomes

$$\begin{aligned} \underset{x\in\mathcal{R}^n, y\in\mathcal{R}^T}{\text{minimize}} \quad & f_0(x_1,\ldots,x_T) \\ \text{subject to} \quad & a_t^\top x_t + y_t \leq b_t,\ \forall t = 1,\ldots,T \\ & y_t \geq y_{t-1} \\ & y_t \geq D_tx_t,\ \forall t = 1,\ldots,T \end{aligned} \tag{A.3}$$

If nonnegative slack variables $\{v_t\}_{t=2}^T$ are used to express the time coupling constraints, (A.3) can be transformed into (A.2).

□

Corollary 2. *Let* $\chi \subseteq \mathcal{R}^{nT\times 1}$ *be a convex solution set of the decision variables* $x = [x_1;\ \ldots;\ x_T]^\top$ *and* f_0 *a convex function. The convex optimization problem*

$$\begin{aligned} \underset{x\in\mathcal{R}^n}{\textit{minimize}} \quad & f_0(x_1,\ldots,x_T) + \max\left\{[D_1x_1;\ldots;D_Tx_T]^\top\right\} \\ \textit{subject to} \quad & x \in \chi \end{aligned} \tag{A.4}$$

is equivalent to

$$\begin{aligned} \underset{x\in\mathcal{R}^n, w\in\mathcal{R}}{\textit{minimize}} \quad & f_0\left(x_1,\ldots,x_T\right)+w \\ \textit{subject to} \quad & x\in\chi \\ & y_T \leq w \\ & D_t x_t \leq y_t,\ \forall t=1,\ldots,T \\ & y_t - y_{t-1} - v_t = 0,\ \forall t=2,\ldots,T \end{aligned} \tag{A.5}$$

Proof. Using epigraph concept of the second term in the objective function, problem (A.4) is expressed as

$$\begin{aligned} \underset{x\in\mathcal{R}^n, w\in\mathcal{R}}{\text{minimize}} \quad & f_0(x)+w \\ \text{subject to} \quad & x\in\chi \\ & \max\left\{\left[D_1x_1;\ \ldots;\ D_Tx_T\right]^\top\right\}\leq w \end{aligned} \tag{A.6}$$

With lemma 1, problem (A.6) is converted into (A.5).

□

APPENDIX B. ACRONYMS

AARC	affinely adjustable robust counterpart
AC	adaptation cost
ACT	ddvanced combustion turbine
AEO	Anual Energy Outlook
ARO	Adjustable RO
BIO	biomass
BTU	British thermal unit
CEP	Capacity expansion planning
CO	Dual unit advanced pulverized coal
CO_2	carbon dioxide
CO_2^{cap}	carbon cap
D	demand
DCOPF	direct current optimal power flow
DDP	dual dynamic programming
DP	dynamic programming
EENS	Expected energy not served
EIA	Energy Information Administration

ENS Energy not served

EENSP Expected energy not served percentage

ENSP Energy not served percentage

ERP Expected robustness price

FRCC Florida Reliability Coordinating Council

GEO geothermal

GHG greenhouse gas

GP gas price

GPL gas production limits

H High

IGCCCS integrated gasification combined cycle with carbon sequestration

L Low

LDC load duration curve

MC Monte Carlo

MRO Midwest Reliability Organization

MSW municipal solid waste

NGCC natural gas combined cycle

NGCCCS natural gas combined cycle with carbon sequestration

NPCC Northeast Power Coordinating Council

NUC nuclear

O&M Operation and Maintenance

OPF optimal power flow

OWND offshore wind

PF Perfect foresight

PoR price of robustness

RC robust counterpart

RFC Reliability First Corporation

RO Robust Optimization

RP robustness price

RPS Renewable Portafolio Standards

SA Sensitivity Analysis

SERC SERC Reliability Corporation

SP Stochastic Programming

SPP Southwest Power Pool

SUN solar thermal

TRE Texas Regional Entity

WAT hydro

WC wind investment cost

WECC Western Electricity Coordinating Council

WND wind

LIST OF SYMBOLS

α^0	Independent coefficient of the voltage angle rule
α^d	Coefficient that multiplies d of the voltage angle rule
$\alpha^{F^{\max}}$	Coefficient that multiplies $F^{\max}$ of the voltage angle rule
β	Investment and adaptation cost trade-off parameter
β^0	Independent coefficient of the power generation rule
β^{FC}	Coefficient that multiplies FC of the power generation rule
$\boldsymbol{\omega}$	Index for referring to an element of Θ
χ	Representation of global uncertain variable
$\Delta Cap_{i,j,t,\boldsymbol{\omega}}$	Direction of capacity adaptation
η^{FC}	Fuel cost primitive uncertainty vector
η^d	Demand primitive uncertainty vector of
$\eta^{F_{\max}}$	Transmission capacity primitive uncertainty vector
γ^0	Independent coefficient of the investment decision rule
γ^{FC}	Coefficient that multiplies FC of the investment decision rule
$\hat{\chi}$	Representation of local uncertainty of χ under realization $real$
$\hat{a}$	Uncertainty of data vector a
λ	Factor by which each uncertainty $\hat{a}$ is multiplied

$\tilde{\rho}$	Minimum level of renewable penetration
$\tilde{\mathbf{g}}^{\max}$	Maximum natural gas production
$\widetilde{\mathbf{CO}}^{\text{cap}}$	Carbon cap
$\mathcal{F}$	Fuel set
$\mathcal{L}$	Transmission path set
$\mathcal{S}$	LDC step set
$\mathcal{T}$	Time stage set
$\mathcal{U}$	General uncertainty set
$\mathcal{Z}$	Primitive uncertainty set
Ω	General uncertainty budget
Ω^{CC}	Uncertainty budget in credited capacity constraint
Ω^{CF}	Uncertainty budget in capacity factor constraint
Ω^{d}	Uncertainty budget in demand constraint
Ω^{FC}	Uncertainty budget in fuel cost
Ω^{Inv}	Uncertainty budget in investment cost
$\Omega^{\text{O\&M}}$	Uncertainty budget in O&M cost
Ω^{obj}	Uncertainty budget in objective function
$\overline{z}$	Upper bound of the objective function in DDP algorithm
Φ	Region set
π	General Lagrange multiplier
Ψ	Technology set

Ψ_{F}	Fuel-based technology set
Ψ_{NF}	Nonfuel-based technology set
σ	Objective function protection term (risk)
Θ	Scenario set formed by all the combinatios of each global uncertainty
θ	Voltage angles
$\underline{z}$	Lower bound of the objective function in DDP algorithm
ζ	Discount factor
A	General data matrix
a	General uncertain data vector
B	General data matrix
b	Line susceptances
c	General cost vector
C_{inv}	Investment cost
C_{op}	Operational cost
Cap	Installed capacity
Cap^{add}	Capacity investment or additions
Cap^{existing}	2009 installed capacity
Cap^{ret}	Capacity retirements
Cap^{f}	Flexible capacity
$Cap^{\boldsymbol{\omega}}$	Adapted capacity
CC	Capacity credit

CF	Capacity factor
d	Demand
DNS	Demand not served
E	General data matrix
$E^{\mathrm{CO_2}}$	Amount of CO_2 emissions per unit of electric energy produced
F	General data matrix
f	Element of $\mathcal{F}$
$F_{l,t}^{\max}$	Transmission capacity
FC	Fuel cost
FOR	Forced outage rate
G	General data matrix
H	Heat rate
h	Duration of the LDC steps
$h^{\mathrm{CC\ acum}}$	Slack variable of credited capacity constraints
h^{CC}	Slack variable of nonfuel based credited capacity constraints
$h^{\mathrm{CF\ acum}}$	Slack variable of capacity factor constraints
h^{CF}	Slack variable of nonfuel based capacity factor constraints
h^{risk_1}	Slack variable of fuel cost risk constraints 1
h^{risk_2}	Slack variable of fuel cost risk constraints 2
h^{risk}	Slack variable of objective function risk constraints
I	Unit investment cost

i, k, n	Element of Φ
j	Element of Ψ
l, l'	Element of $\mathcal{L}$
m	Element of Ψ_f
OM^{f}	Fixed O&M costs
OM^{v}	Variable O&M costs
P	Power generation
p	Future objective function in DDP algorithm
q	General decision vector
R	Unit retirement cost
r	Reserve requirement
S	Incidence matrix of the system
s, s', v	Elements of $\mathcal{S}$
S^{base}	System base power
SV	Salvage value
T	Planning horizon
t, t', τ	Element of $\mathcal{T}$
TC	Total cost
u	Element of Ψ_{NF}
x	General decision vector
x^f	General flexible solution vector

x_0	Initial condition
y	General decision vector
z	General protection term
$z^{2\ \mathrm{res}}$	Auxiliar protection term of reserve constraints
$z^{\mathrm{CC\ acum}}$	Protection term of credited capacity constraints
z^{CC}	Protection term of nonfuel-based credited capacity constraints
$z^{\mathrm{CF\ acum}}$	Protection term of capacity factor constraints
z^{CF}	Protection term of nonfuel-based capacity factor constraints
z^{Inv}	Protection term of minimum investment constraints
z^{res}	Protection term of reserve constraints
z^{d}	Protection term of nodal power balance constraints
$z^{\theta_{\max}}$	Protection term of maximum voltage angle constraints
$z^{\theta_{\min}}$	Protection term of minimum voltage angle constraints
$z^{F_{\max}}$	Protection term of maximum transmission capacity constraints
$z^{F_{\min}}$	Protection term of minimum transmission capacity constraints
$z^{P_{\min}}$	Protection term of nonfuel-based minimum power generation constraints

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